Vol. XLI. JUNE, 1899.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

No. 850.

THE DETERMINATION OF THE SAFE WORKING STRESS FOR RAILWAY BRIDGES OF WROUGHT IRON AND STEEL.

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PRESENTED AT THE ANNUAL CONVENTION, JULY, 1898.

WITH DISCUSSION.

INTRODUCTORY.

From the date of the first introduction of railways, it was for many years the custom, in the design of a railway bridge, to treat the Moving Load of the train in precisely the same manner as the Fixed Load of the structure itself. The load due to the weight of the train was simply added to the load due to the weight of the structure (with flooring, ballast, permanent-way, etc.), and the section of iron to be used calculated for a working stress of about one-fourth of the assumed breaking stress. This, for wrought iron in tension, allowed a nominal unit stress of 5 tons per square inch.

Later on, text-books, dealing theoretically with the subject, taught that the effect produced by Live Load was just double that produced by Dead Load, and recommended that the two kinds of load should be treated separately; a factor of safety of 6 being used with the former and 3 with the latter. Taking the breaking stress of wrought iron

in tension as 21 tons per square inch,* the safe working stress per square inch was given as—

Dead Load	7	tons.
Live Load	3½	"

It was not recognized that the theoretical "Live Load," on which these considerations were based, was one which was never dealt with in practice, and was, moreover, essentially different in its action and mode of application from that due to a locomotive at speed. Still this system showed a great advance on what had gone before, and had the effect of directing attention to the matter. The result was that, although engineers generally regarded this as a theoretical rule to which it would not be prudent to work fully in practice, it became common to limit the stress on tension bars to 4 tons per square inch for live load, and it was generally acknowledged that for dead load the stress per square inch in tension might safely be taken at 6 tons. The Board of Trade limit of 5 tons, however, precluded any improvement in the latter direction.

This may be regarded as the first recognition of the principle that Moving Load may be equated with Fixed Load by the use of a suitable coefficient; but at this stage the coefficient was supposed to be always the same, irrespective of the relative amounts of the two kinds of load to be dealt with.

As wrought-iron bridges of large span became more common, engineers grew to be familiar with the fact that, with the comparatively great mass of metal in a large-span bridge, the effect of a train is relatively less than with a small-span bridge. In view of this fact it became usual in the practice of some of the leading bridge engineers to adopt a scale by which the unit stress allowed was made to vary with the span. It was thus acknowledged that the effect of Moving Load as compared with Fixed Load was not always the same, but became relatively less as the proportion of Fixed Load to Moving Load became greater.

The principle of graduating or apportioning the unit stress to the relative amount of Moving Load work to be done soon became gener-

* For the purposes of this paper it is assumed that the static breaking stress per square inch, for material as ordinarily used for bridge girders, may be taken as 21 tons for wrought iron and 27 tons for steel. Also, that the safe working stress may be taken as one-third of the breaking stress. Hence, for an entirely static or fixed load, the figures adopted are as follows:

	Iron.	Steel.
Breaking stress in tons per square inch	21.00	27.00
Safe working stress in tons per square inch	7.00	9.00

ally recognized; and in America the application of this principle to practical purposes received the most careful study by many of the bridge engineers of that country. Under the special allotment system, as developed by American engineers, the members of a bridge truss are grouped or classified according to the relative amount of Moving Load work they have to bear, depending on the span of the bridge and the position of the member in the truss. To each group is then assigned a percentage by which the nominal stress (treated as static) is to be increased to obtain the equivalent effective working stress; the permissible stress per unit of area remaining constant. In more recent specifications similar results are obtained by using the nominal stress (treated as static), and allotting to each group a different permissible stress per unit of area.

Meantime the results of the magnificent series of experiments by Wöhler and Bauschinger had been ably dealt with by Launhardt, Weyrauch, Gerber and others, and bridge engineers began to consider the cumulative effect of frequent repetitions of the Moving Load.

Thus, while forty years ago a ton of Moving Load received no more consideration than a ton of Fixed Load, its disposal now is recognized as involving very complex considerations. It is found that for any member of a bridge the immediate effect of the application of a ton of Moving Load is always greater than that produced by a ton of Fixed Load, and that the degree or percentage by which it is greater varies for different ratios of Moving Load to Fixed Load. Similarly, that for repeated loading and unloading, the ultimate cumulative effect, on any member of a bridge, of a ton of Moving Load, is always greater than the immediate effect, and that the degree or percentage by which it is greater varies for different ratios of the initial stress or Fixed Load.

The Nature of the Effects.

Definitions.—The terms "Dead Load" and "Live Load" have already been appropriated in a certain definite sense by the text-books; and in this sense have a special relative value, the effect of Live Load being exactly double that due to Dead Load. For the sake of distinction therefore in this paper the terms "Fixed Load" and "Moving Load" will be used with meanings respectively as follows:

Fixed Load.—By this term is to be understood a load which may for practical purposes be considered absolutely constant, subject to no

movement, vibration or variation. The self-supported weight of the structure itself is thus throughout this paper treated as "Fixed Load" and also any other load supported by the structure which is stationary or which generally remains in position and is not frequently applied or removed. Thus, for example, the weight of the girders of a bridge would be treated as Fixed Load, so would also the ballast, sleepers, permanent-way, etc.*

Moving Load.—By this term is to be understood a load which is subject to variation or which is frequently applied and removed. The effect of the violence or vibration with which the application of the load is accompanied is considered as an enhancement of the effect due to the mere weight of the load. Thus a dead engine, pulled over a bridge by a rope from a crab winch beyond, would be treated as Moving Load; the same locomotive traversing the bridge at full speed would be considered as having an enhanced effect due to the violence and vibration with which its application was accompanied.

The "Live Load" of the Text-Books.—In the case of a girder, the "Live Load" of the text-books would mean a load suddenly applied in a vertical direction, but without impact. This might be represented by a load just touching the upper surface of the girder, but having its weight entirely supported by a cord. If that cord were instantaneously severed, the load would then act as "Live Load" in the ordinary text-book meaning of the term. The same load, unsupported by the cord, and having its whole weight resting on the girder without vibration, would represent "Dead Load."

It can easily be shown that the effect on the girder of the "Live Load" of the text-books is theoretically double the effect of the corresponding "Dead Load."

Live Load Contrasted with Train Load.—Those writers who apply the theoretical "Live Load" of the text-books to the solution of this problem, assume that the effect produced by the train traversing the bridge in a horizontal direction may be taken as the dynamic equivalent of a load suddenly applied in a vertical direction; and that the faster the engine moves, the more nearly does it become "Live Load," in the text-book meaning of the term.

* For the purpose of experiment on the relative effect of Fixed Load and Moving Load, an engine standing on a bridge is considered as Fixed Load, the same engine traversing the bridge at full speed being taken as Moving Load.

In certain cases with alternating stresses the weight of the structure or a portion of the same may become "Moving Load," *i. e.*, a load applied and removed each time an engine traverses the bridge.

It will, however, be evident on consideration that these two modes of applying the stress are in every respect essentially different. In the case of the "Live Load" of the text-books a load is applied vertically by the steady action of gravity, while in the case of the Moving Load on a railway bridge there is a locomotive engine running horizontally by means of its own self-developed power, with all the plunging and peculiar vibration contingent on the special construction of the machine, the irregularities of the permanent-way, and the deflection of the girders.

An engine standing at rest on a bridge is certainly "Dead Load" in the ordinary text-book meaning of the term; but when that engine begins to move, the effect on the bridge, due to gravity, is reduced instead of being increased. If the effects of vibration and deflection be neglected, and the engine be supposed to run with absolute smoothness, it is clear that the faster the engine moves in a horizontal direction, the less will be its effect in a vertical direction, and that if the speed became infinite the effect due to gravity would become nothing.

The only way in which a train passing on to a bridge in a horizontal direction could be made approximately to represent a load suddenly applied in a vertical direction would be to cause the train to run at a great speed on to the bridge and stop suddenly in the middle.

It is further to be noted that whereas the "Live Load" of the text-book rule means a load suddenly applied, but without shock or violence, the Moving Load of a locomotive differs essentially therefrom; its effect being less on one hand, in that it is not suddenly applied, while it is greater on the other hand, in that its application is always accompanied by a certain amount of vibration, which is in some cases of great violence.

It is evident, therefore, that a consideration of the "Live Load" of the text-books will not help in any way toward a knowledge of the effect produced on a bridge by the Moving Load of a locomotive. The two kinds of load are essentially different in their nature and in their mode of application; the effect produced by the locomotive may be greater than that of the theoretical "Live Load," or it may be less, and should it in any case be found by experiment to be the same, this can only be regarded as a coincidence.

Two-Fold Effect of Moving Load.—When a locomotive at full speed traverses a bridge, there is an immediate measurable effect produced,

which is greater than that due to the same load at rest. This immediate extra effect is due to the sudden and violent manner in which the load is applied, the result being an increased deflection of the structure as a whole, accompanied by a general jarring and vibration, which is especially noticeable in the smaller and lighter members.

It has, moreover, been established, by the experiments of Wöhler, Bauschinger and others, that the immediate effect of the Moving Load is by no means the whole effect, but that frequent repetitions of stress have a cumulative effect on the structure, of very great importance; and it has been ascertained that a bar subjected to stress, repeatedly applied and removed, will ultimately break with a load very much less than it would have borne for a few applications only.

Thus there are two distinct effects of the Moving Load to take into account, and it appears necessary to draw special attention to this point, as it is one which is often lost sight of, even by good authorities.

On one hand it is sometimes found that the "Cumulative Effect" is ignored, and it is argued that on a large-span bridge the effect of the Moving Load is practically no greater than that of the same weight as Fixed Load, because the observed extra deflection is but trifling. On the other hand it is frequently found that the "Immediate Effect" is ignored, and the deductions from Wöhler's experiments by Launhardt, Weyrauch and others are discussed as if cumulative effect were the only matter for consideration.

To determine a coefficient for the Moving Load by a study of cumulative effect alone would be to assume the effect on a bridge of an engine traveling at sixty miles an hour to be no more severe than that due to the same engine "dead" hauled over slowly with a rope from a winch beyond the abutment. On the other hand to determine a coefficient for the Moving Load by a study of the immediate effect alone, would be to assume the effect on a bridge of an engine traversing the structure an indefinite number of times to be no greater than that produced by the engine traversing the structure once only.

In this paper, therefore, the effect of the Moving Load will be considered under two heads, viz.: (*a*) Immediate Effect—which is observable every time a train passes over the bridge; and (*b*) Cumulative Effect—produced in course of time by repeated loading and unloading.

Immediate Effect of the Moving Load.

The Measure of Immediate Effect.—If a locomotive be run slowly on to a bridge and there brought to a stand, the stresses produced will be simply those due to the weight of the machine as Fixed Load. If the engine be now made to traverse the bridge at a considerable speed, the immediate effect of the extra stress produced is indicated by an increased deflection of the structure as a whole, and by an increased elongation or shortening of members in tension or compression.

These effects may be measured by suitable instruments, and the immediate effect produced by the locomotive in motion can be compared with that produced by the mere weight of the machine at rest. Thus, for example, if the deflection of a girder with the engine at speed is found to be half as much again as that due to the dead weight of the engine at rest, evidently this same extra deflection could have been produced with the engine standing on the bridge by loading it with pig iron until the weight resting on its wheels had been increased by 50 per cent.* A ton of Moving Load in this case being found to produce on the bridge an immediate measurable effect as severe as one and a half tons of Fixed Load, it is clear that for the purpose of determining the stresses the actual weight of the Moving Load would have to be multiplied by a coefficient of 1.5, to obtain its equivalent in terms of Fixed Load.

Observations in India on Deflection.—Since the year 1879 deflection observations have been regularly made in India by the Government Inspector on the bridges of each new section of railway before it is opened for traffic. These observations are, whenever practicable, made with self-recording apparatus, and are conducted on a uniform system under Government rules. The author has made an analysis of all the results thus obtained for ten years, from June, 1882, to June, 1892. This deals with nearly 1 500 separate observations made by a number of different inspecting officers in all parts of India where railways had been constructed during that period, and with every variety of girder in ordinary use, over about 7 500 miles of new railway. Some of these lines are on the 5 ft.-6 in. gauge, others on the meter-gauge. Some are owned by the Government, others by companies.

It will be seen, therefore, that the results thus obtained represent averages, not only of a considerable number of actual observations,

* This, of course, depends on the assumption that the measurable deflection increases directly as the actual stress.

but also for a variety of different spans and different designs. Further, that the experiments were conducted on a uniform system and under practical conditions by a number of entirely independent observers. No doubt, in many cases the original data may have been but roughly determined, and the observers perhaps sometimes did their work in a careless or perfunctory manner; but any errors due to defective observation are as likely to be on one side as the other, and with the large number of observations on which the averages were taken, and the large number of different observers who contributed to these averages, it is probable that the results obtained fairly represent the tendency of the actual effects produced under ordinary conditions.

Collation of the Results.—In collating and arranging these data the average of all the observations was first taken for each span separately, and the results so obtained plotted by points on the diagram. A note was then made at each point to show the number of observations on which the position of that point depended, and each point in succession was joined to that immediately beyond it by a straight line. On each straight line so obtained a mark was made dividing it into two parts in the inverse ratio of the numbers at its end points. These marks thus again averaged the averages previously obtained, in the ratio of their relative importance. It will be seen, therefore, that under this system each point was given votes (as it were) in proportion to the number of observations it represented, and the position in which the curve was finally drawn was determined accordingly. Through the last point, thus obtained experimentally for the greatest proportion of Fixed Load, the line was continued as an even curve to pass through the point of all "Fixed Load" where the immediate effect of the Moving Load becomes nothing. The result thus obtained is illustrated in the diagram, Fig. 1.

Endeavors were made, by different systems of plotting and in other ways, to discover some law or rule by which the form of the curve might be determined, but without success. A parabola was, of course, first suggested, but was found quite irreconcilable by any reasonable concessions. The result now presented, therefore, embodies no theory; it is merely an even curve averaging the actual results as nearly as practicable.

Results Accepted for Practical Purposes.—To what extent the results now obtained for girders as complete structures apply to individual

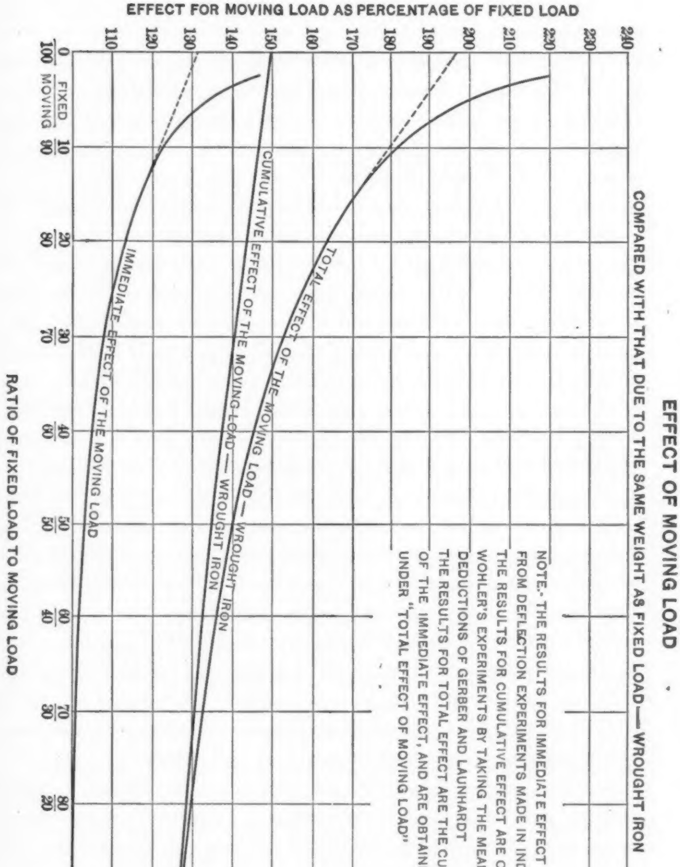
EFFECT OF MOVING LOAD
COMPARED WITH THAT DUE TO THE SAME WEIGHT AS FIXED LOAD—WROUGHT IRON
EFFECT FOR MOVING LOAD AS PERCENTAGE OF FIXED LOAD
110 120 130 140 150 160 170 180 190 200 210 220 230 240
NOTE.- THE RESULTS FOR IMMEDIATE EFFECT ARE DEDUCED FROM DEFLECTION EXPERIMENTS MADE IN INDIA
THE RESULTS FOR CUMULATIVE EFFECT ARE OBTAINED FROM WOHLER'S EXPERIMENTS BY TAKING THE MEAN BETWEEN THE DEDUCTIONS OF GERBER AND LAUNHARDT
THE RESULTS FOR TOTAL EFFECT ARE THE CUMULATIVE EFFECT OF THE IMMEDIATE EFFECT, AND ARE OBTAINED AS EXPLAINED UNDER "TOTAL EFFECT OF MOVING LOAD"
TOTAL EFFECT OF THE MOVING LOAD—WROUGHT IRON
CUMULATIVE EFFECT OF THE MOVING LOAD — WROUGHT IRON
IMMEDIATE EFFECT OF THE MOVING LOAD
FIXED MOVING
0/100 10/90 20/80 30/70 40/60 50/50 60/40 70/30 80/20 90/10 100/0
FIXED MOVING
RATIO OF FIXED LOAD TO MOVING LOAD

Fig. 1.

truss members can only be determined by an extended series of detailed experiments on individual members in girders of different types; but as the law by which the effects are governed must be the same in both cases, it is probable that the scale by which the value of the coefficient varies, for different ratios of Moving Load to Fixed Load, will be of a similar character; and as the results obtained from the deflection of the truss, as a whole, must to a great extent represent the average for all its members, it will be evident that the error cannot in any case be of great importance.

It is probable that a complete series of experiments, carefully conducted with efficient apparatus, would indicate that each class of members of a triangulated girder has its own proper series of coefficients, and that another series would apply to plate girders. If the lines representing each of these series of coefficients were plotted on a diagram, a maximum and minimum boundary line could then be drawn, enclosing a strip or band within which all the coefficient lines would lie.

Remembering that herein only the coefficient for immediate effect is being discussed—that this has to be combined with the cumulative effect to obtain the total effect—that this total effect is for Moving Load alone, and that to obtain the total effective stress on the bar the Fixed Load is to be added—remembering, further, that the safe working stress on a bar, due to all these effects combined, is only one-third of the ultimate breaking stress for that bar—it will, no doubt, be agreed that the width of the strip or band, containing the various coefficient lines, is not likely to be great enough to affect to any important extent the actual dimensions which would be given to any individual member of a truss as a result of these calculations. Still less would it be likely to affect the approximate formula recommended for general use.

For practical purposes, therefore, and in the absence of more complete data, it is considered that the results of the deflection experiments made in India may be accepted as a sufficiently close approximation to the average immediate effect produced on the members of a bridge truss by a passing train.

The general results for the immediate effect of the Moving Load, as obtained from these experiments, are given in Table No. 1*, for different

* For the purposes of this paper it is assumed that the static breaking stress per square inch, for material, as ordinarily used for bridge girders, may be taken as 21 tons for wrought iron and 27 tons for steel. Also, that the safe working stress may be taken as one-third of the breaking stress. Hence, for an entirely static or fixed load, the figures adopted are as follows:

	Iron.	Steel.
Breaking stress in tons per square inch....................	21.00	27.00
Safe working stress in tons per square inch..............	7.00	9.00

conditions of loading, from "All Moving Load" advancing by 5% differences up to "All Fixed Load." These results are also exhibited in a graphic form in Fig. 1.

TABLE No. 1.—IMMEDIATE EFFECT OF MOVING LOAD. DEDUCED FROM DEFLECTION EXPERIMENTS MADE IN INDIA.

Nature of Compound Load. Ratio Percentage.		Immediate effect of Moving Load compared with that of Fixed Load. Percentage.	Immediate Breaking Stress for the Compound Load. Tons per Square Inch.		
			For the Total Load.	Apportionment of the Breaking Stress.	
Fixed Load.	Moving Load.			Due to Fixed Load.	Due to Moving Load.
0	100	......	.....	....	.....
2.5	97.5	147.40	14.36	0.36	14.00
5	95	133.78	15.90	0.79	15.10
10	90	122.63	17.45	1.74	15.70
15	85	116.86	18.37	2.76	15.61
20	80	113.06	19.01	3.80	15.21
25	75	110.27	19.50	4.87	14.62
30	70	108.13	19.87	5.96	13.91
33.3	66.7	107.14	20.05	6.68	13.37
35	65	106.63	20.13	7.05	13.09
40	60	105.47	20.33	8.13	12.20
45	55	104.62	20.48	9.22	11.26
50	50	103.87	20.60	10.30	10.30
55	45	103.23	20.70	11.38	9.31
60	40	102.63	20.78	12.47	8.31
65	35	102.10	20.85	13.55	7.30
66.7	33.3	101.96	20.86	13.91	6.95
70	30	101.64	20.90	14.63	6.27
75	25	101.21	20.94	15.70	5.23
80	20	100.82	20.97	16.77	4.19
85	15	100.53	20.98	17.84	3.15
90	10	100.30	20.99	18.89	2.10
95	5	100.13	21.00	19.95	1.05
100	0	100.00	21.00	21.00	0.00

CUMULATIVE EFFECT OF THE MOVING LOAD.

The celebrated experiments of Wöhler, followed by further observations in the same direction by Spangenberg, Bauschinger and B. Baker, have furnished engineers with a mass of data regarding the cumulative effect produced on a structure by repeated loading and unloading.

The experiments, it must be remembered, were conducted with apparatus specially arranged to apply and remove the load in a steady and uniform manner, the effect of shocks and the violent jarring and hammering action, which is observed when a locomotive traverses a bridge at speed, being entirely eliminated. The results of these ex-

periments on cumulative effect are, therefore, exactly what is requred to supplement the observations on immediate effect.

The general results are familiar to all engineers who have interested themselves in the subject, and are briefly as follows:

Let t = The greatest stress the bar will bear under a static load. Condition — "All Fixed Load."
(*Tragfestigkeit*, or Statical breaking strength.)

u = The greatest stress of which the bar will bear an indefinite number of repetitions with the load applied and removed. Condition — "All Moving Load."
(*Ursprungsfestigkeit*, or Primitive strength.)

$+v$ and $-v$ = The greatest stress of which the bar will bear an indefinite number of alternations. The load producing tension and compression alternately; the stresses being equal, but in opposite directions.

a = The actual stress under which the bar breaks—sometimes called the "Working Strength" of the bar.
(*Arbeitsfestigkeit*, or Ultimate working strength.)

Min. S = The least stress to which the bar is subjected.

Max. S = The greatest stress to which the bar is subjected.

$\phi = \frac{\text{Min. } S}{\text{Max. } S}$. This expression also represents the ratio of initial stress or Fixed Load to the Total Load. For example, if a compound load be made up of one-fourth Fixed Load and three-fourths Moving Load, the value of ϕ will be 0.25.

First.—With a Moving Load applied and removed an indefinite number of times, and alternating from zero to a certain fixed quantity, a bar will ultimately break with a load considerably less than that which it would have been able to bear as Fixed Load. In other words, u is always considerably less than t. The ratio of $u : t$ is found to vary with different materials, and some of the experiments show great discrepancies; but, in a general way, it appears that the difference between u and t is greater in steel with a comparatively large percentage of carbon and high static breaking stress, and less with mild steel having a lower static breaking stress. With wrought iron, moreover, it is generally less than with mild steel.

The actual relative value of u, as compared with t, may be taken roughly as varying from $u = \frac{2}{3} t$ for wrought iron down to $u = \frac{1}{3} t$ for tool steel.

Second.—With a stress alternating from a positive amount, $+ v$, to an equal negative amount, $- v$, the bar will ultimately break if $v =$ about $\frac{1}{2} u$.

Third.—With a certain initial stress or Fixed Load (Min. S), and a certain Moving Load added and removed (causing the total stress to vary by loading and unloading from Min. S to Max. S), the stress on the bar becomes compound, being partly due to Fixed Load and partly due to Moving Load. Under these conditions, the total stress a under which the bar ultimately breaks will vary with the ratio or percentage of Fixed Load to Moving Load.

The extreme limits are: on one side "All Fixed Load," when

Range, Min. S to Max. $S = 0$, then $a = t$ and $\phi = 1$;

and on the other side, "All Moving Load," when

Range, Min. S to Max. $S = u$, then $a = u$ and $\phi = 0$.

Between these limits the value of a for any ratio of Fixed Load to Moving Load can be determined by means of "Gerber's parabola," or by the use of Launhardt's formula.*

Gerber's Parabola.—The range of stress which a bar will bear for an indefinite number of repetitions of the load applied and removed having been determined by Wöhler's experiments for different conditions of loading, it was found by Gerber that if the ranges of stress be plotted as ordinates, and the corresponding minimum stresses as abscissas, the points would fall approximately on a parabolic curve.

Let $\triangle =$ The range of stress, *i. e.*, Max. $S \mp$ Min. S.†

$t =$ The greatest stress the bar will bear under a static load.

$k =$ A constant for the material.

Then Gerber's equation may be written thus—

$$(\text{Min. } S + \tfrac{1}{2} \triangle)^2 + k\triangle = t^2$$

The ultimate static breaking stress for wrought iron, as ordinarily used for bridge girders, may be taken as 21 tons per square inch. For iron of this quality, the results of Wöhler's experiments show that the

* For an account of other methods of dealing with the subject, which have been proposed by Schäffer, Müller, Winkler, Cain, Smith, Seefehlner, Ritter, Lippold, and Clericetti, with tabular comparisons of results, see the paper by Weyrauch published in *Proc. Inst. C. E.*—1882-83.—Vol. lxxi, p. 298.

† The upper sign is to be taken where the stresses are of the same kind and the lower, if of different kinds (*i. e.*, ranging between tension and compression).

greatest stress of which the bar will bear an indefinite number of repetitions with the load applied and removed, will be about 14 tons, *i. e.*, about two-thirds of the ultimate static breaking stress. The corresponding value of k in Gerber's equation will then be 28.

The general results for cumulative effect of the Moving Load, obtained by the use of Gerber's Parabola for wrought iron, are given in Table No. 2, for different conditions of loading from "All Moving Load," when :

Range from Min. S to Max. $S = u$, and $a = u$;

advancing by 5% differences up to "All Fixed Load," when,

Range from Min. S to Max. $S = 0$, and $a = t$.

TABLE No. 2.—CUMULATIVE EFFECT OF MOVING LOAD DEDUCED FROM WÖHLER'S EXPERIMENTS BY GERBER'S PARABOLA.

Nature of Compound Load. Ratio Percentage.		Cumulative effect of Moving Load compared with that of Fixed Load. Percentage.	Ultimate Breaking Stress for the Compound Load. Tons per Square Inch.		
			For the Total Load.	Apportionment of the Breaking Stress.	
Fixed Load.	Moving Load.			Due to Fixed Load.	Due to Moving Load.
0	100	150.00	14.00	0.00	14.00
2.5	97.5	149.00	14.21	0.36	13.86
5	95	148.00	14.42	0.72	13.70
10	90	146.00	14.85	1.49	13.37
15	85	144.00	15.28	2.29	12.99
20	80	142.01	15.72	3.14	12.57
25	75	140.05	16.15	4.04	12.11
30	70	138.12	16.58	4.97	11.60
33.3	66.7	136.85	16.86	5.62	11.24
35	65	136.22	17.00	5.95	11.05
40	60	134.37	17.41	6.96	10.45
45	55	132.57	17.81	8.01	9.80
50	50	130.81	18.20	9.10	9.10
55	45	129.12	18.57	10.21	8.36
60	40	127.48	18.92	11.35	7.57
65	35	125.91	19.25	12.52	6.74
66.7	33.3	125.40	19.36	12.91	6.45
70	30	124.40	19.57	13.70	5.87
75	25	122.96	19.86	14.90	4.97
80	20	121.58	20.13	16.11	4.03
85	15	120.26	20.38	17.32	3.06
90	10	119.00	20.61	18.55	2.06
95	5	117.81	20.81	19.77	1.04
100	0	116.67	21.00	21.00	0.00

Launhardt's Formula.—The formula proposed by Launhardt to express the results claimed by Wöhler's experiments is as follows:

$$a = u\left(1 + \frac{t-u}{u} \cdot \frac{\text{Min. } S}{\text{Max. } S}\right)$$

It will be observed that if the results obtained by the use of Launhardt's formula be plotted on the same system as that adopted by Gerber, the curve obtained will also be a parabola.

Taking the data for wrought iron as before (see under Gerber's parabola) there results:

For an entirely static load (*i. e.*, All Fixed Load) $t = 21$;

For a load alternating from zero to maximum (*i. e.*, All Moving Load) $u = 14$.

Let $\frac{\text{Min. } S}{\text{Max. } S}$ be represented by the symbol ϕ; then, for wrought iron, Launhardt's formula may be written thus:

$$a = 14 \left(1 + \frac{\phi}{2}\right)$$

The general results for the cumulative effect of the Moving Load, obtained by the use of Launhardt's formula for wrought iron, are given in Table No. 3 for different conditions of loading from "All Moving Load," when:

Range from Min. S to Max. $S = u$, then $a = u$ and $\phi = 0$, advancing by 5% differences up to "All Fixed Load," when:

Range from Min. S to Max. $S = 0$, then $a = t$ and $\phi = 1$.

Total Effect of the Moving Load.

Definition of Total Effect.—The total extra effect produced by the Moving Load, as compared with that due to the same weight as Fixed Load, may be taken as that which would be produced by an indefinite number of repetitions of the immediate effect. In other words, the total effect which the bridge should be designed to bear with safety is the ultimate cumulative effect of the immediate effect.

Method of Calculation.—Let it be assumed, for example, that a certain member of a bridge girder has to be designed to bear a compound load, of which 10% is due to Fixed Load, and 90% due to Moving Load, giving a ratio of—

$$\frac{10 \text{ fixed}}{90 \text{ moving}}$$

and that the actual amount of the Moving Load is 100 tons.

This 100 tons of Moving Load is, owing to the violence with which it is applied, found by experiment (see Table No. 1) to produce an immediate effect as severe as that which would be produced by 122.63 tons applied quietly.

TABLE No. 3.—CUMULATIVE EFFECT OF MOVING LOAD. DEDUCED FROM WÖHLER'S EXPERIMENTS BY LAUNHARDT'S FORMULA.

Nature of Compound Load. Ratio Percentage.		Cumulative effect of Moving Load compared with that of Fixed Load. Percentage.	Ultimate Breaking Stress for the Compound Load. Tons per Square Inch.		
			For the Total Load.	Apportionment of the Breaking Stress.	
Fixed Load.	Moving Load.			Due to Fixed Load.	Due to Moving Load.
0	100	150.00	14.00	0.00	14.00
2.5	97.5	149.38	14.18	0.35	13.82
5	95	148.78	14.35	0.72	13.63
10	90	147.62	14.70	1.47	13.23
15	85	146.51	15.05	2.26	12.79
20	80	145.45	15.40	3.08	12.32
25	75	144.44	15.75	3.94	11.81
30	70	143.48	16.10	4.83	11.27
33.3	66.7	142.86	16.33	5.44	10.89
35	65	142.55	16·45	5.76	10.69
40	60	141.67	16.80	6.72	10.08
45	55	140.82	17.15	7.72	9.43
50	50	140.00	17.50	8.75	8.75
55	45	139.22	17.85	9.82	8.03
60	40	138.46	18·20	10.92	7.28
65	35	137.74	18.55	12.06	6.49
66.7	33.3	137.50	18.67	12.44	6.22
70	30	137.04	18.90	13.23	5.67
75	25	136.36	19.25	14.44	4.81
80	20	135.71	19.60	15.68	3.92
85	15	135.09	19.95	16.96	2.99
90	10	134.48	20.30	18.27	2.03
95	5	133.90	20.65	19.62	1.03
100	0	133.33	21.00	21.00	0.00

Having found the enhancement of stress due to the violence with which the load is applied, the cumulative effect will evidently be that due to an indefinite number of repetitions of this enhanced stress. For cumulative effect, therefore, the load which is applied to and removed from the member every time the train traverses the bridge must be taken as 122.63 tons (instead of 100 tons). The Fixed Load, however, remains as before.

Hence, for the purpose of determining the cumulative effect, the ratio—

$$\frac{10 \text{ fixed}}{90 \text{ moving}}$$

will be changed, the Moving Load being increased by 22.63%, and the ratio will become—

$$\frac{10 \text{ fixed}}{110.367 \text{ moving}}$$

or as a percentage ratio—

$$\frac{8.308 \text{ fixed}}{91.692 \text{ moving}}$$

The coefficient for ultimate cumulative effect, applicable to this ratio by Gerber's Parabola, is 1.467. This coefficient is to be applied to 122.63 tons (not 100 tons) and—

$$122.63 \times 1.467 = 179.9 \text{ tons.}$$

By Launhardt's formula the results would be:

$$122.63 \times 1.480 = 181.5 \text{ tons.}$$

Mode of Application—Hence with a ratio of $\frac{10 \text{ fixed}}{90 \text{ moving}}$, a Moving Load of 100 tons applied and removed an indefinite number of times would produce a total effect (the cumulative effect of the immediate effect) equivalent to about 180 tons as fixed load. In other words, with this ratio, to obtain the equivalent of the Moving Load in terms of Fixed Load, the coefficient would be 1.8; and, to determine the dimensions of the member, the Moving Load may be multiplied by 1.8, the result added to the actual Fixed Load, and, for the purposes of calculation, the total so obtained used as "All Fixed Load."

Statement of Results.—The general results for the total effect of the Moving Load (*i. e.*, the ultimate cumulative effect of the immediate effect) for wrought iron are given in Table No. 4 for different combinations of loading, from "All Moving Load," advancing by 5% differences, up to "All Fixed Load." The total effect here given is calculated as explained above, the cumulative effect as used for calculation being taken as the mean between the results obtained by the use of Gerber's Parabola and those obtained by the use of Launhardt's formula. These results are also exhibited in a graphic form in Fig. 1.

Results Applied to Practice.

Results by Experiment.—For wrought iron, the actual effects for a Compound Load, as determined by experiment, for different combinations of Fixed Load and Moving Load, are shown in Table No. 4.

It will be remembered that these figures represent the combined results for immediate effect and for cumulative effect, and that for immediate effect the curve was not plotted by any rule, but was merely

TABLE No. 4.—TOTAL EFFECT OF MOVING LOAD. THE ULTIMATE CUMULATIVE EFFECT OF THE IMMEDIATE EFFECT.

NATURE OF COMPOUND LOAD. RATIO PERCENTAGE.		Total effect of Moving Load compared with that of Fixed Load. Percentage.	ULTIMATE BREAKING STRESS FOR THE COMPOUND LOAD. TONS PER SQUARE INCH.		
			For the Total Load.	Apportionment of the Breaking Stress.	
Fixed Load.	Moving Load.			Due to Fixed Load.	Due to Moving Load.
0	100	......	......	......	......
2.5	97.5	220.31	9.66	0.24	9.42
5	95	199.04	10.82	0.54	10.28
10	90	180.68	12.17	1.22	10.95
15	85	170.42	13.14	1.97	11.17
20	80	163.16	13.95	2.79	11.16
25	75	157.44	14.68	3.67	11.01
30	70	152.75	15.34	4.60	10.74
33.3	66.7	150.30	15.73	5.24	10.48
35	65	149.07	15.92	5.57	10.35
40	60	145.93	16.46	6.59	9.88
45	55	143.31	16.96	7.63	9.33
50	50	140.90	17.43	8.72	8.72
55	45	138.70	17.89	9.84	8.05
60	40	136.63	18.32	10.99	7.33
65	35	134.71	18.73	12.17	6.55
66.7	33.3	134.13	18.85	12.57	6.29
70	30	132.94	19.11	13.38	5.73
75	25	131.28	19.48	14.61	4.87
80	20	129.73	19.82	15.86	3.96
85	15	128.36	20.14	17.12	3.02
90	10	127.13	20.45	18.40	2.04
95	5	126.02	20.73	19.69	1.04
100	0	125.00	21.00	21.00	0.00

drawn in evenly, to average, as nearly as practicable, the actual values obtained by experiment. The corresponding curve has, therefore, no equation, and the results can only be utilized by a reference to the table itself.

This process would, however, not be specially troublesome, as, in practice, a corresponding table or diagram would be used to facilitate computations made on any modern system, or based on an ordinary formula, such as that of Launhardt. It would, in either case, merely be necessary to ascertain the percentage ratios of Moving Load to Fixed Load, and take the corresponding figures from the table or diagram.

It is, nevertheless, certainly desirable to have a simple rule or easily remembered formula, and the following are accordingly offered for consideration, as giving results sufficiently near to those obtained by experiment, and as being, at the same time, easy of application. The effect of each of these systems, for wrought iron, is exhibited graph-

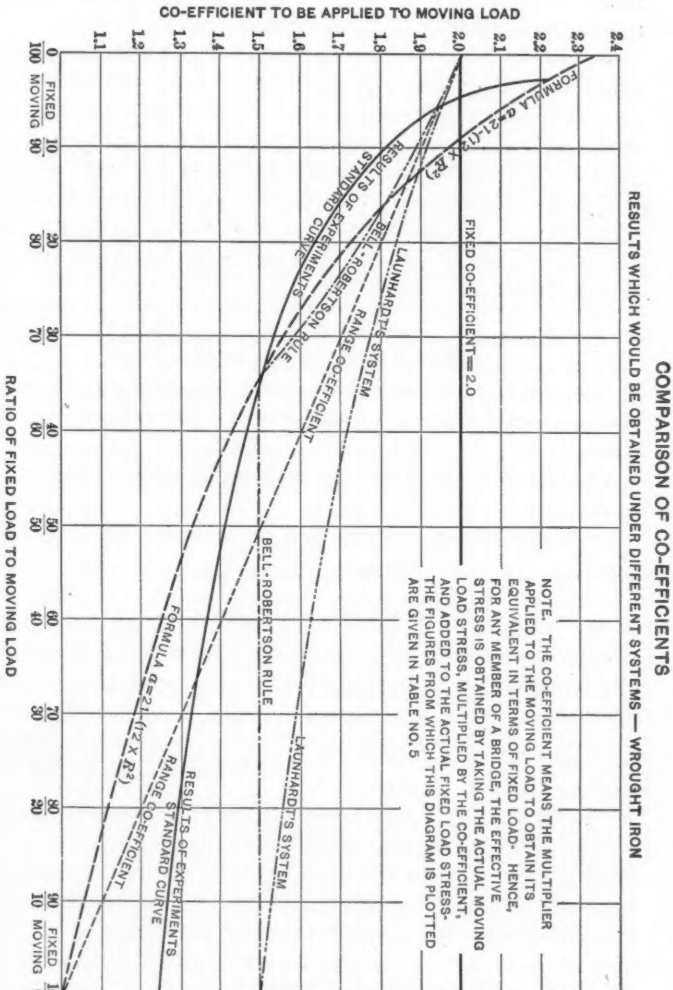
COMPARISON OF CO-EFFICIENTS
RESULTS WHICH WOULD BE OBTAINED UNDER DIFFERENT SYSTEMS — WROUGHT IRON
CO-EFFICIENT TO BE APPLIED TO MOVING LOAD
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
RATIO OF FIXED LOAD TO MOVING LOAD
FIXED
MOVING
NOTE. THE CO-EFFICIENT MEANS THE MULTIPLIER APPLIED TO THE MOVING LOAD TO OBTAIN ITS EQUIVALENT IN TERMS OF FIXED LOAD. HENCE, FOR ANY MEMBER OF A BRIDGE, THE EFFECTIVE STRESS IS OBTAINED BY TAKING THE ACTUAL MOVING LOAD STRESS, MULTIPLIED BY THE CO-EFFICIENT, AND ADDED TO THE ACTUAL FIXED LOAD STRESS. THE FIGURES FROM WHICH THIS DIAGRAM IS PLOTTED ARE GIVEN IN TABLE NO. 5
FIXED CO-EFFICIENT = 2.0
FORMULA $a = 2.1 - (1.2 \times R^2)$
RESULTS OF EXPERIMENTS
STANDARD CURVE
LAUNHARDT'S SYSTEM
BELL-ROBERTSON RULE
RANGE CO-EFFICIENT

Fig. 2.

ically in Figs. 2 and 3, for comparison with one another and with the results of experiment. The curve representing the results of experiment is, for distinction, called the "Standard Curve."

General Principles.—It is assumed as established, that, for any Compound Load, the variation in the breaking stress per unit of area depends on the range of stress; that is to say, on the relative amount of Moving Load in the Compound Total Load. To assist in the investigation of this subject, and to enable different systems to be properly compared, it will, therefore, be convenient to adopt a symbol which will directly represent the range. For stresses all in the same direction (*i. e.*, all in tension or all in compression): Range of stress = Moving Load = Max. S — Min. S.

$$\text{Let } R = \frac{\text{Moving Load}}{\text{Moving Load} + \text{Fixed Load}} = \frac{\text{Range}}{\text{Total}}$$

Then the value of R increases directly as the range of stress, and represents the ratio which the range of stress bears to the total stress.

The extreme limits in the value of R are, on one side, "All Fixed Load," when:

$$\text{Min. } S = \text{Max. } S, \text{ then } a = t \text{ and } R = 0,$$

and on the other side, "All Moving Load" when:

$$\text{Min. } S = 0, \text{ then } a = u \text{ and } R = 1.$$

It will be observed that R is thus the complement of the symbol used for the Launhardt formula, thus :—

$$R + \phi = 1,\ R = 1 - \phi,\ \phi = 1 - R$$

and, remembering this relation, it is easy to translate an expression from one system to the other.

Fixed Coefficient.—Under this system each ton of Moving Load is assumed to have a constant relative value, as compared with a ton of Fixed Load, and, on any member of a bridge, the nominal unit stress would merely depend upon how much Moving Load and how much Fixed Load had to be compounded at that relative value.

It will be seen, however, from the results of experiment, that the effect of Moving Load, as compared with that of Fixed Load, is not in all cases the same, but is relatively greater as the proportion of Moving Load becomes higher. Thus, according to the results of experi-

ment, the coefficients proper to different conditions of loading are as follows:

Moving Load.	Fixed Load.	Actual Coefficient.*
10	90	1.27
50	50	1.41
90	10	1.81

It might, therefore, at first sight appear that under the single coefficient system it would not be possible to obtain a graduated scale for the permissible unit stress which should be fairly in accord with the requirements of the case; but it will be found on consideration that under this system the graduation is better than might at first be supposed; and that a single coefficient, if selected as applicable to a high ratio of Moving Load to Fixed Load, would in itself provide a sliding scale yielding results which would perhaps nowhere differ to an important extent from those which would be obtained under a more complete system.

Remembering that the coefficient is applied only to that part of the Total Load which has been called "Moving Load," it is evident that its effect in increasing the size of any member becomes relatively small as the proportion of "Moving Load" to be taken by that member is reduced. Thus, with a single coefficient selected as applicable to a high ratio of Moving Load as compared with Fixed Load, the percentage of error in the Total Load obtained by its use would be decreased roughly in proportion as the actual error in the coefficient itself became greater; and with the maximum error in the coefficient, as in the condition of (practically) all Fixed Load, the error in the Total Load used for the determination of the dimensions of a member would become (practically) nil.

* As there has been some misunderstanding with regard to this coefficient, the following illustration is offered with an apology to those who, being well acquainted with the matter, will no doubt regard it as superfluous:

Suppose that a number of bullets, all of the same size, be made, some of copper and some of gold; and that each copper bullet weighs one ounce.

Suppose further, in the first instance, that the specific gravity of gold be exactly double that of copper. Under these conditions, with any mixed lot of bullets—some of copper and some of gold—it is evident that the weight of the lot in ounces could at once be ascertained by counting the number of each kind separately, multiplying the gold number by two, and adding the result to the actual copper number. This represents the case for a single fixed coefficient.

Suppose now the conditions altered, and that the specific gravity of the gold bullets does not remain the same, but in a mixed lot is found to be higher when the gold bullets are relatively numerous, and lower when the reverse conditions prevail. To obtain the total weight for any mixed lot now, not only would the number of bullets of each metal have to be counted, but the ratio of the numbers would have to be determined, and the calculation be based on a specific gravity for gold which would vary with that ratio. This represents the case for a variable coefficient.

This will be apparent from the following example:

Case 1.—Conditions assumed: Moving Load, 9 tons; Fixed Load, 1 ton; Correct Coefficient, 2.0.

Here the correct equivalent Total Load would be $18 + 1 = 19$ tons. Had the coefficient been wrongly taken at 1.5, the equivalent Total Load would have worked out to $13.5 + 1 = 14.5$ tons, showing an error in the result nearly proportioned to the error in the coefficient, and all on the wrong side.

Case 2.—Conditions assumed: Moving Load, 1 ton; Fixed Load, 9 tons; Correct Coefficient, 1.3.

Here the correct equivalent Total Load would be $1.3 + 9 = 10.3$ tons. Had the coefficient been wrongly taken at 2.0, the equivalent Total Load would have worked out to $2 + 9 = 11$ tons, showing an error in the result very small as compared with the error in the coefficient, and all on the side of safety.

This system, with a fixed coefficient of 2.0, has found much favor in America, and has been adopted by several bridge engineers, notably by Theodore Cooper, M. Am. Soc. C. E. It has the advantage of very great simplicity, and gives results which show a very fair approximation to those obtained by experiment.

The corresponding formula is:

$$\text{Safe working stress in tons per square inch} = 7 \times \left(\frac{1}{1 + R}\right)$$

It will be observed on reference to Fig. 3, that the greatest percentage divergence from the standard curve occurs at the point where $\frac{\text{Fixed Load} = 40}{\text{Moving Load} = 60}$; here the results with a fixed coefficient of 2.0 would give an excess of strength of about 25 per cent.

In Fig. 2, for the coefficient, this system is represented by a straight line parallel to the datum line. In Fig. 3, for safe working stress, the bend of the curve is downward toward the datum line throughout, whereas the bend of the standard curve is in the opposite direction, with curvature increasing with the ratio of Moving Load. This peculiarity causes a considerable difference in the direction of the two curves where the ratio of Moving Load is high. The curve here in fact bends the wrong way.

Rule Adopted in India.—It will be observed that, where a fixed coefficient of 2.0 is adopted for all members of a truss, there will be an excess of strength for those members in which the Total Load is made

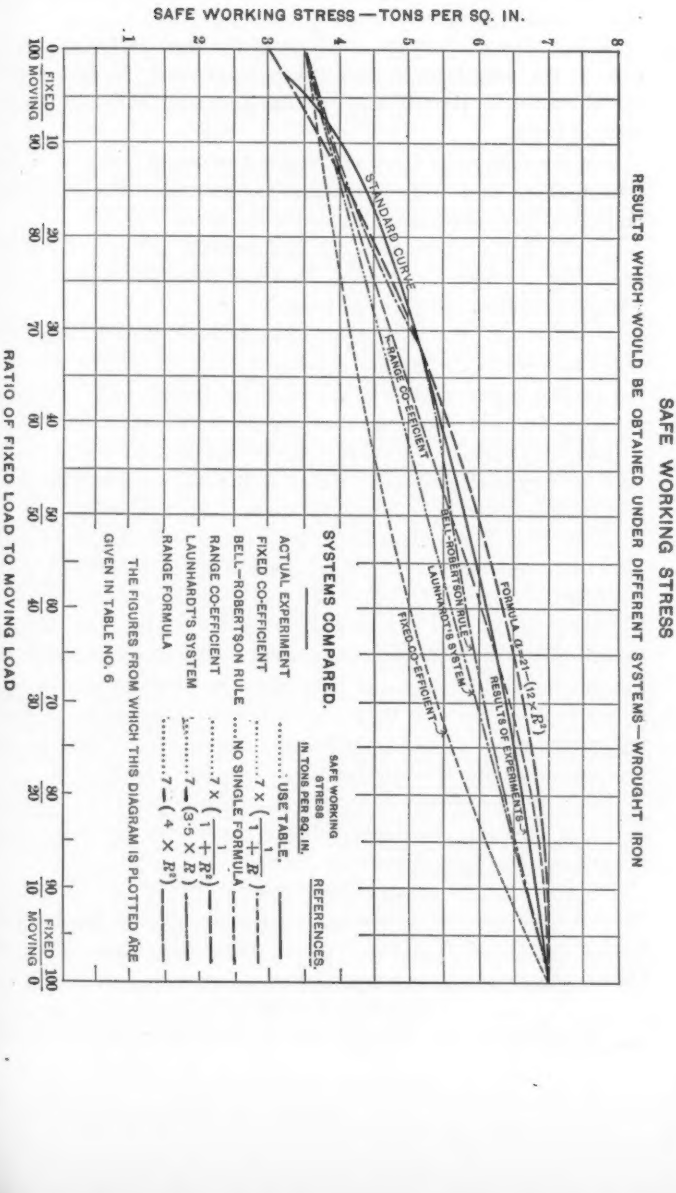
SAFE WORKING STRESS
RESULTS WHICH WOULD BE OBTAINED UNDER DIFFERENT SYSTEMS—WROUGHT IRON
SAFE WORKING STRESS—TONS PER SQ. IN.
0 .1 2 3 4 5 6 7 8
STANDARD CURVE
RANGE CO-EFFICIENT
BELL-ROBERTSON RULE
LAUNHARDT'S SYSTEM
FIXED CO-EFFICIENT
FORMULA $a = 21 - (12 \times R^2)$
RESULTS OF EXPERIMENTS
SYSTEMS COMPARED.
SAFE WORKING STRESS IN TONS PER SQ. IN.
REFERENCES.
ACTUAL EXPERIMENT .... USE TABLE.
FIXED CO-EFFICIENT .... $7 \times \left(\frac{1}{1+R}\right)$
BELL-ROBERTSON RULE ....NO SINGLE FORMULA
RANGE CO-EFFICIENT .... $7 \times \left(\frac{1}{1+R^2}\right)$
LAUNHARDT'S SYSTEM .... $7 - (3.5 \times R)$
RANGE FORMULA .... $7 - (4 \times R^2)$
THE FIGURES FROM WHICH THIS DIAGRAM IS PLOTTED ARE GIVEN IN TABLE NO. 6
$\frac{\text{FIXED}}{\text{MOVING}}$ $\frac{0}{100}$ $\frac{10}{90}$ $\frac{20}{80}$ $\frac{30}{70}$ $\frac{40}{60}$ $\frac{50}{50}$ $\frac{60}{40}$ $\frac{70}{30}$ $\frac{80}{20}$ $\frac{90}{10}$ $\frac{100}{0}$ $\frac{\text{FIXED}}{\text{MOVING}}$
RATIO OF FIXED LOAD TO MOVING LOAD

Fig. 8.

up of a large proportion of Fixed Load. This will be specially observable in the main booms of bridges of large span, for which a coefficient of 1.5 would be more suitable.

With a view to retain the simplicity of the fixed coefficient system, and, at the same time, obtain results more nearly in accord with actual conditions, the following rule has been adopted by the Government of India:

For any member of a railway bridge of wrought iron or steel, the total working load is to be taken as the greatest "Moving Load" multiplied by a coefficient and added to the actual "Fixed Load."

The coefficient to be used for this purpose is 2.0 in all cases, except for the upper and lower booms of triangulated girders, for which a coefficient of 1.5 may be used.

It will be seen that, although two coefficients are used, there will be no step or kink in the line representing the rule graphically. In the graphic representation of the rule there would be two separate lines, one for main booms and the other for general work.

Under the operation of the rule the coefficient of 1.5 would not be used for very small girders, inasmuch as in modern practice in India triangulated girders are not used for spans of less than 80 ft.

It has been shown that, with only a single coefficient, the adjustment of the unit stress, to suit varying ratios of Fixed Load to Moving Load, can be arranged for fairly well, provided the coefficient selected be that applicable to a condition of approximately "All Moving Load." With the addition of the second coefficient for main booms, the provisional rule adopted in India gives results which are probably as nearly in accord with the actual conditions as could be attained without the use of a variable coefficient. The rule has, moreover, the practical advantage of being simple and very easily applied.

This rule is practically in accord with that advocated by T. Claxton Fidler in his able and interesting exposition of his "Dynamic Method."* The formula given by Fidler is

$$\Omega = \text{Max. } S + \omega$$

where Ω represents the momentary internal stress and ω the dynamic increment. For cross-girders, vertical suspenders, diagonals of web bracing and girders up to a span of 20 ft.

$$\omega = \text{Max. } S - \text{Min. } S;$$

* "A Practical Treatise on Bridge Construction," by T. Claxton Fidler, M. Inst. C. E. Edition 1893; p. 260.

this implies that the "effective stress" due to the Moving Load will be obtained by adding a "dynamic increment" equal to the Moving Load. In other words, the Moving Load is to be doubled to obtain its equivalent in terms of Fixed Load.

For main booms of girders of 100 ft. span or upward

$$\omega = \frac{\text{Max. } S - \text{Min. } S}{2};$$

this implies that the "effective stress" due to the Moving Load will be obtained by adding a "dynamic increment" equal to half the Moving Load. In other words, the Moving Load is to be increased by half as much again to obtain its equivalent in terms of Fixed Load.

Bell-Robertson Rule.—This rule was proposed by Messrs. J. R. Bell and F. E. Robertson as an improvement to the Government of India rule given above; the object being to obtain a simple means of varying the coefficient according to the nature of the load.

The rule is as follows:

For any member of a railway bridge of wrought iron or steel, the total "working load" is to be taken as the greatest "live load" multiplied by a coefficient of 1.5 and added to the actual "dead load," provided that a minimum dead load equal to half the live load shall be taken into calculation wherever the actual dead load is less.

Hence, under this rule, the coefficient would be 2.0 for the condition of All Moving Load, and would decrease as the relative amount of Moving Load became less, until at the point $\frac{\text{Moving Load} = 66.7}{\text{Fixed Load} = 33.3}$, the coefficient would become 1.5. From this point onward there would be a fixed coefficient of 1.5.

This rule gives a good approximation to the results as obtained by experiment, and is at the same time simple and easy of application. It is, however, not a uniformly consistent rule, and, if represented graphically, will be seen to be made up of two curves of different characters, with their intersections at the point $\frac{\text{Moving Load} = 66.7}{\text{Fixed Load} = 33.3}$. There is thus a cusp or kink in the line at this point. (See Fig. 2.)

Range Coefficient.—With a coefficient increasing with the range of stress (directly as $1 + R$), a very good approximation is obtained. On this system it is assumed that the extra effect of the Moving Load, as compared with the same weight as Fixed Load, may, for practical pur-

poses, be taken as the nominal Moving Load multiplied by R. The coefficient to be applied to the Moving Load, to obtain its equivalent in terms of Fixed Load, will then be $1 + R$.*

The practical application of this rule is extremely simple, as the coefficient for any compound load is obtained at once by merely adding 1.0 to the decimal giving the proportion of Moving Load to Total Load.

Example:

Moving Load 0.9 of Total Load.

Then coefficient = 1.9.

Moving Load 0.66 of Total Load.

Then coefficient = 1.66.

The corresponding formula is:

$$\text{Safe working stress in tons per square inch} = 7 \times \left(\frac{1}{1+R^2}\right)$$

In Fig. 2, for the coefficient, this system is represented by a straight line drawn from the point 2.0 for "All Moving Load," to 1.0 for "All Fixed Load."

In Fig. 3, for safe working stress, the curve representing this system has a double (or reversed) curvature. It will be seen that it gives a very good approximation to the standard curve throughout.

Launhardt's System.—Launhardt's formula is:

$$a = u\left(1 + \frac{t-u}{u} \cdot \frac{\text{Min. } S}{\text{Max. } S}\right)$$

Avoiding troublesome fractions, the nearest approximation to the results for total effect, obtained by experiment, will be found by assigning to u a value of one half t. For wrought iron, t being taken as 21 tons, the formula would then become—

$$\text{Breaking stress in tons per square inch} = 10.5 \times (1 + \phi);$$

and with a factor of safety of 3.0—

$$\text{Safe working stress in tons per square inch} = 3.5 \times (1 + \phi)$$

It is to be noted that this result is the same as that which has been arrived at in America by an entirely independent course of reasoning, and is now adopted in some of the best modern American specifications.

Using the symbol R instead of ϕ, the formula would be written thus:

$$\text{Safe working stress} = 3.5\,(2 - R)$$
$$= 7.0 - 3.5\,R.$$

* As a peculiarity incidental to this system, it may be noted that for any Compound Load in which Fixed Load + Moving Load = 100, the equivalent (or effective) Total Load will be $100 + (10\,R)^2$. For example, $\frac{\text{Fixed Load} = 20}{\text{Moving Load} = 80}$, then $R = 0.8$, and the effective Total Load is 164.

The use of this formula, with the values adopted, involves the following assumptions:

(*a*) The coefficient to be applied to the Moving Load, to obtain its equivalent in terms of Fixed Load, is 2.0 for the condition of "All Moving Load." diminishing as the ratio of Moving Load to Fixed Load decreases, until it becomes 1.5 for the condition of "All Fixed Load." The variation in the value of the coefficient is directly in the ratio of Total Load to Fixed plus Total Load. (See Fig. 2.)

(*b*) The nominal breaking stress varies directly as the ratio of Fixed Load to Total Load. Hence, in Fig. 3, for "nominal" safe working stress, this system is represented by a straight line.

This formula is very simple in application and has the advantage of being already well known and established in practice in America. It will also be seen by a reference to Fig. 3 that the results obtained on this system give a fairly good approximation to the results obtained by experiment.

It will be observed, however, that, being a straight line formula (see Fig. 3), it is impossible by any adjustment of the relative values of u and t to obtain results for safe working stress decreasing in a higher ratio as the percentage of Moving Load becomes greater. To give such results, and thereby be more in accord with the facts as indicated by experiment, the line as plotted in Fig. 3 should bend upward, *i. e.*, be convex as viewed from the top of the diagram.

Range Formula.—It being established that the breaking stress for a bar becomes less as the range of stress increases, it is evident that a useful formula may be constructed under which the breaking stress for a compound load may be arrived at by means of an expression which will show directly, as a function of the range, the amount by which the static breaking stress is to be reduced. On the system now proposed, therefore, the static breaking stress will be the standard for comparison in all cases, and the lower breaking stress for any compound load will be ascertained by a direct subtraction of the amount by which the breaking stress is reduced.

Avoiding troublesome fractions and complicated expressions, it is considered that for wrought iron, with a static breaking stress of 21 tons per square inch, the most suitable formula for general use would be:

Breaking stress, in tons per square inch, $= 21 - (12 \times R^2)$; and with a factor of safety of 3:

Safe working stress, in tons per square inch, $= 7 - (4 \times R^2)$

In Fig. 2, for the coefficient, the results obtained by the use of this formula are represented by a line extending from 2.33 for "All Moving Load," to 1.00 for "All Fixed Load."

In Fig. 3, for safe working stress, it will be observed that for the lower ratios of Moving Load, the working stress which would be allowed under this formula is somewhat higher than that given by the standard curve. The greatest percentage of difference is at the point where the Moving Load is about half the Fixed Load — here, under the formula, the permissible working stress per square inch would be 6.56 tons, as against 6.28 tons indicated as suitable by the results of experiment. The line represented by this formula crosses the standard curve near the point where the Moving Load is about double the Fixed Load; and thence for the higher ratios of Moving Load the working stress which would be allowed under the formula is lower than that indicated by the experimental results. At the point where the compound load becomes "All Moving Load," the permissible working stress, as given by the formula, is 3.0 tons per square inch.

General Summary.—The six systems, which have now been compared and discussed, for giving approximately the safe working stress for wrought iron, are:*

(*a*) *Actual Experimental Results:*

Safe working stress obtained by use of table.

(*b*) *Fixed Coefficient:*

Safe working stress in tons per square inch $= 7 \times \left(\frac{1}{1 + R}\right)$

(*c*) *Bell-Robertson Rule:*

No single formula applicable.

(*d*) *Range Coefficient:*

Safe working stress in tons per square inch $= 7 \times \left(\frac{1}{1 + R^2}\right)$

(*e*) *Launhardt's System:*

Safe working stress in tons per square inch $= 7 - (3.5 \times R)$

(*f*) *Range Formula:*

Safe working stress in tons per square inch $= 7 - (4 \times R^2)$

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* In these formulas the symbol R represents the proportionate range of stress, thus:

$$R = \frac{\text{Moving Load}}{\text{Fixed Load + Moving Load}} = \frac{\text{Range}}{\text{Total}}$$

The "Safe Working Stress" in each case is "Nominal Stress" due to the weight of the Moving Load and Fixed Load simply added. The corresponding "Effective Stress" obtained by the use of these formulas is 7 tons per square inch throughout.

TABLE No. 5.—Results for the Coefficient. Wrought Iron.

Nature of Compound Load. Ratio Percentage.		Effect of Moving Load—Percentage. Compared with the Effect Due to the Same Weight Applied as Fixed Load.					
		Results of Experiment for Actual Total Effect.	Fixed Coefficient. 2.0	Bell-Robertson Rule.	Range Coefficient.	Launhardt's System $u = \frac{1}{2} t$.	Range Formula.
Fixed Load.	Moving Load.		$a = 21 \times \left(\frac{1}{1+R}\right)$		$a = 21 \times \left(\frac{1}{1+R^2}\right)$	$a = 21 - (10.5 \times R)$	$a = 21 - (12 \times R^2)$
0	100	....	200.00	200.00	200.00	200.00	233.33
2.5	97.5	220.31	200.00	197.44	197.50	197.56	221.96
5	95	199.04	200.00	194.74	195.00	195.24	212.09
10	90	180.68	200.00	188.89	190.00	190.91	195.74
15	85	170.42	200.00	182.35	185.00	186.96	182.73
20	80	163.16	200.00	175.00	180.00	183.33	172.07
25	75	157.44	200.00	166.67	175.00	180.00	163.16
30	70	152.75	200.00	157.14	170.00	176.92	155.56
33.3	66.7	150.30	200.00	150.00	166.67	175.00	151.06
35	65	149.07	200.00	150.00	165.00	174.07	148.96
40	60	145.93	200.00	150.00	160.00	171.43	143.17
45	55	143.31	200.00	150.00	155.00	168.96	138.00
50	50	140.90	200.00	150.00	150.00	166.67	133.33
55	45	138.70	200.00	150.00	145.00	164.51	129.08
60	40	136.63	200.00	150.00	140.00	162.50	125.16
65	35	134.71	200.00	150.00	135.00	160.60	121.51
66.7	33.3	134.13	200.00	150.00	133.33	160.00	120.34
70	30	132.94	200.00	150.00	130.00	158.82	118.07
75	25	131.28	200.00	150.00	125.00	157.14	114.81
80	20	129.73	200.00	150.00	120.00	155.56	111.70
85	15	128.36	200.00	150.00	115.00	154.05	108.68
90	10	127.13	200.00	150.00	110.00	152.63	105.75
95	5	126.02	200.00	150.00	105.00	151.27	102.86
100	0	125.00	200.00	150.00	100.00	150.00	100.00

It will be observed that, of these systems, each has some advantage to recommend it, and that in the selection of the most suitable system for general use, it is to some extent a question of how far it may be considered advisable to sacrifice simplicity to obtain a closer approximation to the results obtained by experiment.

For convenience of comparison the results which would be obtained by the use of each rule or formula for different ratios of Fixed Load to Moving Load are exhibited in the following tables:

Table No. 5.—Results for the Coefficient.

Table No. 6.—Results for Safe Working Stress.

Table No. 7.—Percentage Comparison.

From the latter table can be ascertained at a glance the practical effect of any rule or formula on the area in cross-section of a member as compared with the area which would be indicated as the result of experiment.

TABLE No. 6.—RESULTS FOR SAFE WORKING STRESS. WROUGHT IRON.

NATURE OF COMPOUND LOAD. RATIO PERCENTAGE.		SAFE WORKING STRESS WITH FACTOR OF SAFETY = 3.0. TONS PER SQUARE INCH.					
		Results of Experiment for Actual Total Effect.	Fixed Coefficient 2.0	Bell-Robertson Rule.	Range Coefficient.	Launhardt's System. $u = \frac{1}{2}t$	Range Formula.
Fixed Load.	Moving Load.		$a = 21 \times \left(\frac{1}{1+R}\right)$		$a = 21 \times \left(\frac{1}{1+R^2}\right)$	$a = 21 - (10.5 \times R)$	$a = 21 - (12 \times R^2)$
0	100	....	3.50	3.50	3.50	3.50	3.00
2.5	97.5	3.22	3.54	3.59	3.59	3.59	3.20
5	95	3.61	3.59	3.68	3.68	3.67	3.39
10	90	4.06	3.68	3.89	3.87	3.85	3.76
15	85	4.38	3.78	4.12	4.06	4.02	4.11
20	80	4.65	3.89	4.38	4.27	4.20	4.44
25	75	4.89	4.00	4.67	4.48	4.37	4.75
30	70	5.11	4.12	5.00	4.70	4.55	5.04
33.3	66.7	5.24	4.20	5.25	4.85	4.67	5.22
35	65	5.31	4.24	5.28	4.92	4.72	5.31
40	60	5.49	4.37	5.38	5.15	4.90	5.56
45	55	5.65	4.52	5.49	5.37	5.07	5.79
50	50	5.81	4.67	5.60	5.60	5.25	6.00
55	45	5.96	4.83	5.71	5.82	5.42	6.19
60	40	6.11	5.00	5.83	6.03	5.60	6.36
65	35	6.24	5.18	5.96	6.24	5.77	6.51
66.7	33.3	6.28	5.25	6.00	6.30	5.83	6.56
70	30	6.37	5.38	6.09	6.42	5.95	6.64
75	25	6.49	5.60	6.22	6.59	6.12	6.75
80	20	6.61	5.83	6.36	6.73	6.30	6.84
85	15	6.71	6.09	6.51	6.85	6.47	6.91
90	10	6.82	6.36	6.67	6.93	6.65	6.96
95	5	6.91	6.67	6.83	6.98	6.82	6.99
100	0	7.00	7.00	7.00	7.00	7.00	7.00

The results are also exhibited graphically for the coefficient in Fig. 2, and for safe working stress in Fig. 3.

With reference to the remarks made on this subject when discussing the fixed coefficient system, it will be observed that where the ratio of Fixed Load, as compared with Moving Load, is high, very great differences in the coefficient, as given in Table No. 5, have but a small effect on the practical results, as given in Tables Nos. 6 and 7.

Approximate Nature of Results.—It is to be remembered that the figures here quoted as "determined by experiment" are merely the averages of a large number of results, among which there are great discrepancies; and that for each point determined the maximum and minimum often differ widely from each other. Further, that in deciding where to place the average value, various circumstances must be taken into account and allowed due weight; but the precise amount of correction required must in each case (within certain limits) remain

TABLE No. 7.—Percentage Comparison. Wrought Iron.

Nature of Compound Load. Ratio Percentage.		Area of Members in Cross-Section. Compared with the Area Indicated by Experiment. Percentage.					
Fixed Load.	Moving Load.	Results of Experiment for Actual Total Effect.	Fixed Coefficient 2.0 $a = 21 \times \left(\frac{1}{1 + R}\right)$	Bell-Robertson Rule.	Range Coefficient. $a = 21 \times \left(\frac{1}{1 + R^2}\right)$	Launhardt's System. $u = \frac{1}{2} t$ $a = 21 - \left(10.5 \times R\right)$	Range Formula. $a = 21 - \left(12 \times R^2\right)$
0	100	....	....	....	....	....	....
2.5	97.5	100	90.89	89.74	89.76	89.79	100.74
5	95	100	100.48	97.89	98.03	98.14	106.39
10	90	100	110.08	104.28	104.86	105.33	107.85
15	85	100	115.74	106.35	107.74	108.79	106.54
20	80	100	119.57	106.29	108.94	110.72	104.74
25	75	100	122.31	104.84	109.21	111.83	103.00
30	70	100	124.16	102.25	108.83	112.36	101.43
33.3	66.7	100	124.81	99.87	108.17	112.33	100.38
35	65	100	125.09	100.46	107.85	112.32	99.95
40	60	100	125.43	101.91	106.62	111.99	98.70
45	55	100	125.17	102.97	105.19	111.40	97.64
50	50	100	124.53	103.78	103.78	110.70	96.86
55	45	100	123.49	104.33	102.42	109.89	96.31
60	40	100	122.11	104.66	101.17	109.02	96.00
65	35	100	120.38	104.77	100.09	108.08	95.88
66.7	33.3	100	119.71	104.78	99.76	107.74	95.87
70	30	100	118.31	104.66	99.20	107.07	95.94
75	25	100	115.93	104.34	98.54	106.00	96.18
80	20	100	113.26	103.83	98.17	104.87	96.60
85	15	100	110.30	103.11	98.08	103.70	97.17
90	10	100	107.09	102.23	98.33	102.48	97.92
95	5	100	103.65	101.18	98.96	101.25	98.86
100	0	100	100.00	100.00	100.00	100.00	100.00

a matter for judgment and discretion. In the interpretation of the results of Wöhler's experiments, for example, it will be seen from Tables Nos. 2 and 3 that such eminent authorities as Gerber and Launhardt are by no means in accord as to the value of the coefficient where the ratio of Fixed Load is high, the coefficient for wrought iron for the condition of "All Fixed Load," according to Launhardt, being one-seventh greater than that obtained by Gerber.

On this subject Weyrauch remarks as follows : *

"Considering that absolutely exact laws for constructive materials will certainly never result from experiments, that even in brands of iron acknowledged to be good, differences in the statical breaking strength t of as much as 40% occur, and that it is merely a question of finding a substitute for the still more rough and incorrect assumption of a constant a, even the preceding might suffice for practical purposes until more facts are accumulated.

* *Min. Proc. Inst. C. E.*, 1880-81, Vol. lxiii, pp. 282 and 283.

"Even for more exact determinations than those under consideration such an approximation ought to be considered satisfactory. To the author's mind it would appear sufficient if the deviations of the real values of a from those given by the formula did not exceed the deviations from one another of real values of a in good and commonly used materials."

If, therefore, by means of a simple rule or easily applied formula a fairly good approximation be obtained to the figures representing the result of experiment, it will evidently not be worth while to adopt a more complicated or troublesome system in order to secure a very close approximation to a line, the correct position of which (within certain limits) is after all a matter of some uncertainty.

The Selection of a System.—In the immediate effect, as determined by experiments, it is to be noted that the only effects allowed for are those which produce a measurable elongation or shortening of a bar or deflection of a beam, and the effect of the violent concussion or vibration, with which the application of the load is accompanied, is not taken into account.

It appears reasonable, however, to suppose that where the application of a load is accompanied by violent jarring or shock, there must be an effect on the bar more severe than that indicated by the mere temporary elongation, shortening or deflection.* The effect of jarring and vibration would be most severe with members (such as cross-girders and rail-bearers) which are exposed more immediately to the action of the locomotive, and generally with members for which the ratio of Moving Load is high as compared with Fixed Load. Hence, in making allowance for the effect of shocks and violent jarring and vibration, it will be proper to arrange that the allowance shall increase as the ratio of Moving Load to Total Load becomes greater, or, in other words, as the value of R becomes higher.

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* In this connection the following illustrations are offered for consideration :—

Let two precisely similar bars be subjected to repeated deflections, equal in extent, involving stress beyond the elastic limit. Let the deflections of the first bar be produced by steady pressure applied and removed. Let the deflections (equal in extent) of the second bar be produced by successive blows of a hammer. It would be expected that the number of deflections before breaking would be less in the second case than in the first.

Again—Let a glass tube be supported in a vertical position, and closed at the lower end. Into this tube let a certain quantity of dry angular sand, not sufficient to fill the tube, be poured in quietly. Having marked on the tube the height occupied by the sand, let the sand be emptied out and again poured in as before. This second time, however, let the tube be violently jarred and shaken as the sand falls into it. In the second case, the sand will not stand at so great a height in the tube as in the first case.

Again—Let a bar of steel be placed with its axis pointing to the magnetic pole, and be left undisturbed. In the course of a few years it will have become to some extent magnetic. The same bar in that position if violently jarred by the blows of a hammer would have become equally magnetic in the course of a few minutes.

TABLE No. 8.—RESULTS FOR WROUGHT IRON. OBTAINED BY USE OF RANGE FORMULA. BREAKING STRESS $= 21 - (12 \times R^2)$.

Nature of Compound Load. Ratio Percentage.		Breaking Stress for the Compound (nominal) Load. Tons per Square Inch.			Coefficient to obtain equivalent of Moving Load in terms of Fixed Load.	Permissible Working Stress.	
		For the Total Load.	Apportionment of the Breaking Stress.			Factor of Safety = 3.	
Fixed Load.	Moving Load.		Due to Fixed Load.	Due to Moving Load.		Nominal Stress. Tons per Square Inch.	Effective Stress. Tons per Square Inch.
0	100	9.0000	0.0000	9.0000	2.3333	3.0000	7.00
2.5	97.5	9.5928	0.2398	9.3530	2.2196	3.1976	7.00
5	95	10.1700	0.5085	9.6615	2.1209	3.3900	7.00
10	90	11.2800	1.1280	10.1520	1.9574	3.7600	7.00
15	85	12.3300	1.8495	10.4805	1.8273	4.1100	7.00
20	80	13.3200	2.6640	10.6560	1.7207	4.4400	7.00
25	75	14.2500	3.5625	10.6875	1.6816	4.7500	7.00
30	70	15.1200	4.5360	10.5840	1.5556	5.0400	7.00
33.3	66.7	15.6672	5.2224	10.4448	1.5106	5.2224	7.00
35	65	15.9300	5.5755	10.3545	1.4896	5.3100	7.00
40	60	16.6800	6.6720	10.0080	1.4317	5.5600	7.00
45	55	17.3700	7.8165	9.5535	1.3800	5.7900	7.00
50	50	18.0000	9.0000	9.0000	1.3333	6.0000	7.00
55	45	18.5700	10.2135	8.3565	1.2908	6.1900	7.00
60	40	19.0800	11.4480	7.6320	1.2516	6.3600	7.00
65	35	19.5300	12.6945	6.8355	1.2151	6.5100	7.00
66.7	33.3	19.6668	13.1112	6.5556	1.2034	6.5556	7.00
70	30	19.9200	13.9440	5.9760	1.1807	6.6400	7.00
75	25	20.2500	15.1875	5.0625	1.1481	6.7500	7.00
80	20	20.5200	16.4160	4.1040	1.1170	6.8400	7.00
85	15	20.7300	17.6205	3.1095	1.0868	6.9100	7.00
90	10	20.8800	18.7920	2.0880	1.0575	6.9600	7.00
95	5	20.9700	19.9215	1.0485	1.0286	6.9900	7.00
100	0	21.0000	21.0000	0.0000	1.0000	7.0000	7.00

On the other hand, of the irregular or abnormal effects to be covered by the factor of safety, it will be observed that many of these causes of extra unit stress would have but little effect on a large and massive member on which the ratio of Fixed Load is high as compared with Moving Load; and it would therefore appear proper to allow a somewhat higher effective stress* as the ratio of Moving Load to Total Load becomes less, or, in other words, as the value of R becomes lower.

Range Formula Recommended.—These results are attained for wrought iron by the use of the range formula:

Safe working stress in tons per square inch $= 7 - (4 \times R^2)$.

* Effective stress means the actual working stress to which the material is subjected, on the assumption that the coefficient adopted correctly represents the real effects of Moving Load as compared with that of the same weight as Fixed Load. In other words, "Effective Stress" means the stress due to the Fixed Load added to that due to the equivalent of the Moving Load in terms of Fixed Load.

With this formula it will be seen, by an inspection of Table No. 6 and of Fig. 3, that, as compared with the actual results of experiment, a somewhat higher effective stress would be allowed on the more massive parts of a large bridge truss where the irregular and abnormal effects would be least, and where economy of material may most profitably be exercised. On the other hand, with the lighter members of a truss (where the ratio of Moving Load is high as compared with Fixed Load, and where the irregular and abnormal effects would be most severely felt), the effective stress allowed would be somewhat lower than that deduced from the results of experiment.

The general results for wrought iron, obtained by the use of this formula, are exhibited in Table No. 8.

Results for Steel.

It will be observed that the value of u, as determined by Wöhler's experiments for steel, is but little greater than $\frac{1}{2}t$, instead of $\frac{2}{3}t$, the value obtained for wrought iron. The result is that the corresponding breaking stress, in the case of "All Moving Load," as deduced from these experiments, would be but little greater for steel than for wrought iron, although in the case "All Fixed Load" the breaking stress per unit of area for steel is one-third higher. This result would be somewhat unsatisfactory to bridge engineers, as, if acted upon, it would have the effect of requiring the area of a member subjected to a high ratio of moving load to be nearly as great with steel as with iron.*

It is to be noted, however, that much of the steel used in Wöhler's experiments was made about forty years ago.† The elastic limit of steel as now commonly used in bridge-building is probably higher, and there is no difficulty in obtaining a material having an elastic limit as high as two-thirds the ultimate static breaking stress.

Hence it would appear that for the quality of mild steel, as now used for the construction of railway bridges, if the ultimate breaking stress per square inch be taken as 27 tons, the elastic limit may, for practical purposes, be taken as high as 18 tons. The rule for safe

* If the immediate effect of Moving Load be neglected, it would appear from the results of Wöhler's experiments that, under the condition of "All Moving Load," the breaking stress for steel might actually be less than that deduced for wrought iron, thus:

For wrought iron.................................. $u = \frac{2}{3}t$ and $t = 21$
For steel.. $u = \frac{1}{2}t$ and $t = 27$

The breaking stress for "All Moving Load" corresponding to these conditions would be, per square inch:

Wrought iron.. 14.00 tons
Steel.. 13.50 tons

† Wöhler's experiments were carried on for twelve years, from 1859 to 1870.

working stress may then be based on a value of u equal to $\frac{2}{3}t$* and the rule for steel may then take the same general form as that found suitable for wrought iron.

On these considerations the formula recommended for steel is:

Safe working stress in tons per square inch $= 9 - (5 \times R^2)$; corresponding with that proposed for wrought iron.†

The general results for steel, obtained by the use of this formula, are exhibited in Table No. 9.

TABLE No. 9.—RESULTS FOR STEEL. OBTAINED BY USE OF RANGE FORMULA. BREAKING STRESS $= 27 - (15 \times R^2)$.

NATURE OF COMPOUND LOAD. RATIO PERCENTAGE.		BREAKING STRESS FOR THE COMPOUND (NOMINAL) LOAD. TONS PER SQUARE INCH.			Coefficient to Obtain Equivalent of Moving Load in Terms of Fixed Load.	PERMISSIBLE WORKING STRESS. Factor of Safety = 3.	
		For the Total Load.	Apportionment of the Breaking Stress.				
Fixed Load.	Moving Load.		Due to Fixed Load.	Due to Moving Load.		Nominal Stress. Tons per Square Inch.	Effective Stress. Tons per Square Inch.
0	100	12.0000	0.0000	12.0000	2.2500	4.0000	9.00
2.5	97.5	12.7410	0.3185	12.4225	2.1478	4.2470	9.00
5	95	13.4625	0.6731	12.7894	2.0585	4.4875	9.00
10	90	14.8500	1.4850	13.3650	1.9091	4.9500	9.00
15	85	16.1625	2.4244	13.7381	1.7889	5.3875	9.00
20	80	17.4000	3.4800	13.9200	1.6897	5.8000	9.00
25	75	18.5625	4.6406	13.9219	1.6061	6.1875	9.00
30	70	19.6500	5.8950	13.7550	1.5344	6.5500	9.00
33.3	66.7	20.3333	6.7778	13.5555	1.4918	6.7778	9.00
35.	65	20.6625	7.2319	13.4306	1.4719	6.8875	9.00
40	60	21.6000	8.6400	12.9600	1.4167	7.2000	9.00
45	55	22.4625	10.1081	12.3544	1.3673	7.4875	9.00
50	50	23.2500	11.6250	11.6250	1.3226	7.7500	9.00
55	45	23.9625	13.1794	10.7831	1.2817	7.9875	9.00
60	40	24.6000	14.7600	9.8400	1.2439	8.2000	9.00
65	35	25.1625	16.3556	8.8069	1.2086	8.3875	9.00
66.7	33.3	25.3333	16.8889	8.4444	1.1974	8.4444	9.00
70	30	25.6500	17.9550	7.6950	1.1754	8.5500	9.00
75	25	26.0625	19.5469	6.5156	1.1439	8.6875	9.00
80	20	26.4000	21.1200	5.2800	1.1136	8.8000	9.00
85	15	26.6625	22.6631	3.9994	1.0844	8.8875	9.00
90	10	26.8500	24.1650	2.6850	1.0559	8.9500	9.00
95	5	26.9625	25.6144	1.3481	1.0278	8.9875	9.00
100	0	27.0000	27.0000	0.0000	1.0000	9.0000	9.00

* For steel having a higher static breaking stress per square inch than 27 tons, the elastic limit would no doubt bear a lower ratio to the ultimate breaking stress than 2 : 3. But in such case the elastic limit would merely be lower relatively, not lower absolutely. A rule based on a breaking stress per square inch, of 18 tons for "All Moving Load," ranging to 27 tons for "All Fixed Load," would clearly not be less safe for a material, the breaking stress for which might range up to (say) 36 tons for "All Fixed Load." Some of the steel used in Wöhler's experiments showed a static breaking stress of more than 50 tons per square inch.

† In this formula the symbol R represents the proportionate range of stress, thus:

$$R = \frac{\text{Moving Load}}{\text{Fixed Load} + \text{Moving Load}} = \frac{\text{Range}}{\text{Total.}}$$

Rule Proposed for Adoption.

For the determination of the Safe Working Stress on any member of a bridge truss the following rule is proposed.

1. The Safe Working Stress for any member of a bridge truss is to be calculated by the following formulas:

Wrought Iron—

$$\text{Safe Working Stress} = \overset{\text{Tons per square inch.}}{7 - (4 \times R^2).}$$

Steel—

$$\text{Safe Working Stress} = \overset{\text{Tons per square inch.}}{9 - (5 \times R^2).}$$

2. In these formulas—

$$R = \frac{\text{Range of Stress}}{\text{Total Stress.}}.$$

N. B.—The Range of Stress is to be taken as the difference between the lowest and the highest stress to which the bar is subjected.*

The Residual Fixed Load Stress is the balance of the Fixed Load Stress which remains unaffected by the application of the Moving Load, and which is therefore not subject to alternation.

The Total Stress is to be taken as the Residual Fixed Load Stress added to the Range of Stress.†

3. For a member subject to compression the area determined under Rule 1 is to be increased as may be necessary by the use of column formula.

4. For a member subject to a Range of Stress, partly in compression and partly in tension, the area required for the compression stress is to be ascertained under Rule 3, and the area required for compound stress is to be ascertained under Rule 1. The greater of the two areas thus found is to be adopted.

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* Hence, to obtain the Range of Stress, if these two extreme stresses be of the same sign (*i. e.*, both tension or both compression) the less is to be subtracted from the greater; if the two extreme stresses be of opposite signs (*i. e.*, one tension and the other compression) they are to be added.

† Hence, for cases where the range is between two stresses of the same sign (*i. e.* both compression or both tension) the Total Stress will be simply the initial Fixed Load Stress added to the greatest Moving Load Stress.

Where the range is between two stresses of opposite signs (*i. e.*, one compression and the other tension) the Residual Fixed Load becomes nothing, and the Total Stress is equal to the Range of Stress; the condition being that of All Moving Load.

CORRESPONDENCE.

HENRY S. PRICHARD, M. Am. Soc. C. E.—The writer has taken a special interest in this paper for the reason that in the specification he prepared for the use of the engineering department of the New Jersey Steel and Iron Company, as revised in 1895, a system of equating the comparative effect of moving and fixed loads is given, which is identical with the system termed "range coefficient" in the paper. The system as given in the specifications referred to is as follows: Mr. Prichard.

"To provide for impact and vibration an amount is to be added to the strains in each member in accordance with the following formulas:

"$I =$ the amount to be added for impact and vibration.

"$L =$ the combined strains from live load and centrifugal force.

"$D =$ the dead load strain.

"For swing bridges and other movable structures, while in motion only, $I = 0.25\ D$.

"For counter strains $I = L$.

"For floor beam hangers and the riveted connections of the floor system $I = 1.25\ L$.

"For all other cases $I = L \dfrac{L}{L + D}$."

The writer derived this system by starting with Launhardt's formula and afterward modifying it to accord with the results of a study of all data, bearing on the subject, which were readily available. These data, however, did not include the valuable observations of the deflections of the bridges of India.

The independent derivation of this system by the author and the writer is a curious coincidence.

The subject of the paper involves two questions, one of fact and the other of method:

First.—What is the relative effect on a bridge member of the moving and fixed loads as compared with the sectional areas required?

Second.—What is the most practicable method of proportioning the member after the relative effect of the moving and fixed loads has been determined?

In discussing the first question, the author points out very clearly that a moving load causes a greater stress than a fixed load of equal magnitude, and that the repeated application and removal of the moving load makes the resulting stress produce a greater destructive effect than an equal amount of stress from a fixed load. He terms the stress caused by the moving load "the immediate effect," and obtains it by deductions from observations of deflections of bridges on the railways of India. He terms the result of repeated stresses the "cumulative effect," and obtains it by using a mean between Gerber's and Launhardt's deductions from Wöhler's experiments. Launhardt's formula

Mr. Prichard. in a modified form has been and is at present extensively used in America for proportioning bridges.

The modification consists in assuming that the dynamic effect, of which Launhardt's formula is supposed to take no account, is similar to that of repeated applications of the load without impact, and in further assuming that the greatest load which a bar can stand when applied an indefinite number of times, with an impact, such as is usual on railroad bridges, will be one-half the greatest fixed load. In Launhardt's formula, when applied to wrought iron, this allowed repeated load is two-thirds the allowed fixed load. The author, by collating the results of the observations of the deflections of the bridges on the railways of India, and deducing therefrom the average increase in the stresses from moving loads, has given something definite, in place of the arbitrary assumptions just referred to. This portion of the author's work has a great value, entirely separate from his subsequent combination of the results obtained with those of Wöhler's experiments.

This subsequent combination is made in a very logical way, and, if the results of Wöhler's experiments are strictly applicable to bridges, it is a rational one to make.

It was formerly, and is to some extent now, supposed that the seeming weakness of the metal from repeated stresses, as shown by Wöhler's experiments, was due to a general deterioration of the metal.

Professor Fidler contends that the seeming weakness can be accounted for by an increase in the stress from the dynamic action of the load.

Another theory is that there is no general deterioration, but that some of the numerous defects, or micro-flaws, gradually extend their weakening influence in an irregular plane of cross-section which ultimately becomes the plane of rupture, while the metal immediately adjacent to this plane remains, perhaps, wholly uninjured.*

It is probable that there is a weakening from repeated stresses in bridge members as well as increased stresses from the dynamic action of the moving load, but it is also probable that the degree of the weakening is less in them, as considerable intervals of rest occur between the passages of trains, than it was in Wöhler's experiments, where there were no such intervals of rest.

Experiments mentioned by Lord Kelvin † show that an elastic vibrator kept vibrating for several days through a certain range, comes to rest much quicker when left to itself than when set in vibration after having been at rest for several days, and then immediately left to itself.

Further, it is a matter of common experience that metal stretched beyond the elastic limit regains its elasticity after a rest.

* "The Materials of Construction," J. B. Johnson, M. Am. Soc. C. E., p. 537.

† "Elasticity," *Encyclopedia Britannica*.

It seems probable, therefore, that, as regards the influence of repeated stresses, the effect of the moving load has been overestimated by the author. On the other hand, it is probable that he has underestimated the dynamic effect, for which provision should be made because he has dealt with averages. Mr. Prichard.

The dynamic effect will vary greatly under different conditions, such as speed, condition of roadway, conditions of rolling stock, etc.

A structure should be designed for the greatest dynamic effect to which the class of structures to which it belongs may be liable, and not merely for the average, though, of course, the interval between maximums must also be considered in deciding upon unit stresses.

One example of the foregoing proposition should be especially considered. It is known that when there is, from any cause, a coincidence between the periods of the dynamic blows from the load and the vibration of a bridge, there will be accumulative vibration and increased stresses therefrom, in some cases the increase being considerable.*

It is probable that for a large proportion of railroad bridges accumulative vibration either does not occur or is quite small, and that for the remainder it only occurs occasionally. Such being the case, its average effect for a large number of bridges would be quite small as compared with its effect on a particular bridge.

In the present state of knowledge, it can hardly be said in advance that any particular bridge will be free from severe accumulative vibration; hence, to be on the side of safety, all bridges should be designed for the effect of severe accumulative vibration, even though comparatively few are affected, and the average effect small; but it may be admissible to use larger unit stresses than should be used if severe accumulative vibrations were certain to occur at frequent intervals.

The results which the author obtains, and which he terms the "results of experiments," are not quite in accord with his judgment when he recommends a system termed "range formula," differing somewhat therefrom, and points out its comparative advantages. The writer prefers the system of "range coefficients" to that of range formula.

Before discussing the relative merits of the different systems, it should be pointed out that for many bridge members not subject to alternate stresses, the minimum stress is not the stress from the fixed load alone, but this stress minus a counter-stress from the moving load. In applying the modified Launhardt formula, many designers have used the stress from fixed load in place of the minimum. On this account the Pennsylvania Railroad Company has added a note of warning in its specifications.

It does not appear from the paper what the author's views are re-

* "Vibration of Bridges," S. W. Robinson, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. xvi, p. 42.

Mr. Prichard. garding such members. The writer advisedly uses the fixed load stress without distinction, both when it is and when it is not the minimum, in the formula given in his specifications; and in his subsequent discussion of the range coefficient system, to which his formula corresponds, this system must be understood as based on such use of the fixed load stress.

The deflections on whose averages the author bases his "immediate effect" curve (Fig. 1), would naturally, for the great majority of cases, be observed for the span fully loaded. When a span is fully loaded, the proportion of fixed load stress to moving load stress is nearly the same for all the members, and is nearly the same as the proportion of fixed to moving load. For those members, therefore, which attain their maximum stresses when the span is fully loaded, which for a span simply supported are the chords and end posts, the immediate effect obtained would seem to apply as an average. It can be noted from Figs. 1 and 2 that at about $\frac{\text{fixed}}{\text{moving}} = \frac{8}{92}$ the curves of immediate effect and total effect, termed by the author "standard curves," change their direction very suddenly, and that the only system that even approximately follows this change below this point is the range formula system, so that for those members whose maximum stress occurs when the span is fully loaded, the other systems ought not to be used if $\frac{\text{fixed}}{\text{moving}}$ is much less than $\frac{8}{92}$.

Using the Pennsylvania Railroad Company's specifications as a criterion, this would mean that the other systems should not be used for spans much less than 12 ft. As 12 ft. is a very short span, even for a stringer, the other systems would not often be ruled out on this account, and where they are specified the point can readily be covered by a qualifying clause.

The effect of accumulative vibration is occasionally so great for truss spans, even of considerable length, that it seems to the writer, that for moderate and moderately large values of $\frac{\text{fixed}}{\text{moving}}$, the standard curve, the range formula and the Bell-Robertson rule all give results which are too small. On the other hand, the fixed coefficient system and the modified Launhardt formula (referred to by the author as Launhardt's system) give results which seem unnecessarily high. The range coefficient system seems to be the best of any discussed. For very large values of $\frac{\text{fixed}}{\text{moving}}$ the results of the standard curve are probably too great, because for large values the dynamic effect, which is probably underestimated, is small as compared with the supposed weakening effect of repeated stresses, which is probably overestimated.

An examination of Fig. 3, Table No. 6, shows that for large values of $\frac{\text{fixed}}{\text{moving}}$, say, for $\frac{67}{33}$ and greater, the unit stress allowed by the range coefficient system is slightly greater than by the standard curve, that allowed by the range formula system is slightly greater than by the range coefficient system, and those allowed by the other systems are less than by the standard curve. Mr. Prichard.

The writer believes that the unit stresses prescribed by the other systems are lower than the facts seem to justify. He prefers the unit stress allowed by the range coefficient system, however, to the somewhat higher unit stress allowed by the range formula system.

The immediate effect of the moving load will vary with the suddenness with which it is applied, the impact with which it strikes the bridge, the accumulative vibration from the impact and the proportion of fixed to moving load; and these variations in the conditions will depend on the speed, character and condition of the rolling stock, and the length, character and condition of the bridge. In obtaining the immediate effect of the moving load, the author has noted the effect on each bridge considered as a whole only, and has considered only the variation in the proportion of fixed to moving load. Therefore, the result obtained for each particular ratio of fixed to moving load is the result for an average bridge, considered as a whole, for an average of all the other conditions. For bridge members whose maximum and minimum stresses occur when the bridge is only partially loaded, the average of the other conditions for any given proportion of fixed to moving would probably be quite different from the average for a bridge considered as a whole, because a small value of $\frac{\text{fixed}}{\text{moving}}$ can only occur for a bridge, thus considered, when the span is short while for members whose maximum and minimum stresses occur under partial loading a small value of $\frac{\text{fixed}}{\text{moving}}$ can occur when the span is long. For such members for a given $\frac{\text{fixed}}{\text{moving}}$ as the span increases, other conditions being constant, the load is applied more gradually. For very small values of $\frac{\text{fixed}}{\text{moving}}$, say $\frac{8}{92}$ and less, the sudden upward bend in the author's curve of immediate effect is probably due to the fact that for such values for bridges considered as a whole, the spans must be very short, and consequently not only is the moving load applied very suddenly, but the maximum impact comes from one blow of the driving wheels of the engine instead of from the aggregate of blows not simultaneously applied from several different units of the rolling load, as is the case for bridges of long span.

For the foregoing reasons the average immediate effect of the mov-

Mr. Prichard. ing load, for each particular value of $\frac{\text{fixed}}{\text{moving}}$, is less for members whose maximum and minimum stresses occur under partial loads than is indicated in the author's diagram, and if the value of $\frac{\text{fixed}}{\text{moving}}$ is very small it is much less. In view of these facts, the writer considers it wise, in applying the range coefficient system to such members, to use the fixed load stress instead of the absolute minimum, thus obtaining a smaller total effect, and to apply this system for even very small values of $\frac{\text{fixed}}{\text{moving}}$ without qualification.

For short spans, the character of the bridge plays an important part in determining the effect of the moving load. For pin bridges, owing to the play in the joints, the impact is greater than for plate and lattice girder bridges.

The writer has seen short pin spans and pin-connected floor beams in which the pin holes had become seriously elongated.

While it is hardly practicable to take the character of the bridges into account in devising a system for equating the effect of the moving load, it is advisable to limit the use of pins to the connections of fairly good-sized trusses.

There are many things, such as the condition of the bridge, the condition of the rolling stock, the amount of deflection, the amount of camber, etc., which help to determine the effect of the moving load. It is, of course, impossible to assign to each its value and to mathematically estimate their combined effect, but the fact of their existence and the uncertainty as to their effect makes it wise to keep well within what superficially seem to be safe limits, and to consider simplicity of greater value than hair-splitting refinements in selecting a system.

With the relative effects of the moving and static load known or assumed the question arises: What is the best method of proportioning? Too little regard is paid to this question. If the supposed facts as to the stresses were followed exactly, the nominal stress would vary with the dynamic action of the moving load, and the allowed unit stresses would vary with the weakening effect of repeated stresses. Precisely the same result can be accomplished, either by keeping the allowed unit stresses constant and providing for both the dynamic effect of the moving load and the weakening effect of repeated stresses by adding a sufficient amount to the nominal stresses, or the allowed unit stresses may be varied and nothing added to the nominal stresses.

The author's diagram, Fig. 1, shows the coefficients by which the moving load has to be multiplied for the various systems he discusses, if the first method is followed; while Fig. 2 shows the unit stresses to be used in the second method.

Mr. Prichard.

Another excellent illustration and contrast of the two methods is afforded by a comparison of the specifications for bridges of the Pennsylvania Railroad Company with those of the Pennsylvania Lines West of Pittsburg. Both are based on the modified Launhardt formula before referred to, but the Pennsylvania Railroad Company uses the first method of proportioning, while the Pennsylvania Lines West of Pittsburg use the second.

The writer's fourteen years' experience in designing and detailing bridges has given him a decided preference for the first method; that is, the method which keeps constant unit stresses and provides for the total effect of the moving load by increasing the nominal stresses.

If the second method is followed, not only should the allowed unit stresses for the main sections be varied, but the allowed shear, bearing, etc., for the pins and rivets should likewise be correspondingly varied. This fact is so evident that it would seem useless to state it were it not for the fact that many, probably a large majority, of the specifications which vary the unit stresses allowed for determining the sectional areas of the main members, make no variation in the allowed shear and bearing for pins and rivets.

Probably few who have not designed by the second method, and consistently varied the unit stresses throughout, realize the labor involved in detailing by this method. In doing so, for every different member of the bridge, and in some cases, such as the stringers and floor beams for different parts of the same member, different bearing and shearing values for the rivets have to be used. This involves a calculation for each case to determine the allowed units, and, as it is not practicable to have tables for all possible unit bearings and shear, a further calculation to determine the value of the rivets in bearing and shear after the allowed units have been determined.

In using the first method, by which the entire effect of the moving load is provided for by increasing the nominal stresses, the allowed unit bearing and shear for the rivets and pins, as well as the allowed unit stresses for the main section, remain constant, and the work of detailing the connections is very much simplified as compared with what it is in using the second method.

In conclusion, the author is to be congratulated on having produced a paper which can hardly fail to considerably influence the future designing of bridges.

Mr. Thurston.

R. H. Thurston, M. Am. Soc. C. E.—The subject is one of supreme importance, not only to the builder of bridges, but also to every worker in metal, whether in construction of structures or machinery. Perhaps it is, if possible, more important to the latter than to the former; for the shake and jar and pound and inertia stresses, in heavy machinery, bring into the problem, in proportioning their parts, many

Mr. Thurston. forces difficult and often impossible to determine exactly, either in amount, direction or effect.

Investigations in two principal directions are gradually revealing the facts, data and laws upon which depend the resultant with which only the engineer is in the end concerned. The action of the dead load, as distinguished from the live load, is an inertia-effect. The action of repeated loads upon the material affected is a molecular phenomenon. The one is related to the weight and its motion simply, the other to the nature of the material attacked by that weight. The resultant effect, the strain produced and the risk involved by that strain must be anticipated by the engineer engaged in designing the piece, structure or the machine affected. In the railway bridge the load may vary from the sum simply of the dead and the live loads, as when the train is barely moving, to some approximation to the sum of the dead and twice the live load, as where the speed is such that the weight of the train may be taken as falling freely, and as ultimately conveying the stored energy of its fall through the extent of the sag of the bridge to the members thus overloaded, which parts, and the structure as a whole, presumably, must absorb all that kinetic energy before the fall can be arrested. It would seem that, as a matter of prudence, the engineer should assume in all such cases (in machine design certainly, and, perhaps, in the planning of parts of structures like bridges) that this maximum may be anticipated, approximately, on occasion, in the course of the life of the bridge or machine. If this be accepted as a principle of design in such cases, the computation of proportions of parts is to that degree simplified, and all errors will occur on the side of safety.

The effect of repeated stresses has now been so fully investigated that, although this field is undoubtedly still open for much profitable research, it may probably be assumed that enough of fact has been accumulated to give the designer of structures subject to repeated stresses, of the same kind at least, sufficient basis for his computations. The writer is inclined to sum the whole matter up by saying that the old idea, held by some of the most distinguished and ablest of investigators and designers, of making the elastic limit the unit upon which to base a factor of safety, has come to be established by the consensus of opinion of all modern investigators. The action of repeated stresses with reversal, involving, as it apparently does, even with small loads, constantly repeated and permanent disturbances of the molecular relations and arrangements, seems likely to demand prolonged experimental investigation and study of every class of material used in engineering. Yet it is now possible, with the more common materials, and for substantially important work, to establish limits, by reference to the experience of the century, which are those within which safety may be assured, practically, in all cases, even in the case

of machinery, with its constant reversal of stresses and strains. All these limit quantities, it may probably be said without hesitation, are found to be so related to the elastic limit of the material as to make that point the bench-mark of our work. The establishment of the elastic limit thus becomes the main requirement, it would seem, of the engineer's tests of materials, so far as used for purposes of construction of either structures or machinery. That being determined for the substance, and its relation to the breaking load under static stress, the judgment of the designer, guided by experience and research, enables him to proportion parts with entire safety and maximum economy in the use of material. Probably the majority of experienced designers of machinery would be satisfied with some such scale of factors of safety as given in Table No. 10:* Mr. Thurston.

TABLE No. 10.—Factors of Safety, Based on Elastic Limit.

Material.	Load.		Shock or repeated load in machinery.	
	Dead.	Live.		
Wrought iron—"mild" steel.......	1	2	3	Ratio of elastic resistance to working load.
Machinery steel,....................	1.5	3	5	
Tool steel..........................	2	4	6	
Cast iron...........................	3	6	8 to 10	

Where, as in bridge building, the movement of weights of great magnitude proportionally to dead load are to be often dealt with, the more precise methods illustrated by Wöhler, Bauschinger, Launhardt, Weyrauch, and by others since the pioneer work of these men was done, come into play. The paper presents an admirable summary of the investigations in this field to date, and the range formula therein given is, it seems, the best combination of simplicity with accuracy that has yet appeared. No one can feel hesitation in its use or distrust of the results when reduced to practice. The one direction in which further development is to be looked for is apparently the adaptation of the formula to use with all the now considerable variety of bridge material available and, in fact, to some extent used.

Formerly the one material obtainable, practically, for this purpose was wrought iron, which, when of really good quality, was a comparatively uniform and distinctly defined material in all its constructive properties. It had a tenacity of from 48 000 to 52 000 lbs. per square inch, and an elastic limit of not far from 18 000 to 20 000 lbs., and to insure uniformity and reliability of quality inspection could readily be made. To-day, iron is largely displaced by steel, and the quality of this latter constructive material may be made to range from that of a basic steel, reproducing substantially the properties

* "Materials of Engineering," Vol. ii, p. 341. R. H. T.

Mr. Thurston. of the softest grades of Swedish iron, through all the range familiar to the builder as the older standard bridge iron up to the various grades of harder steels, having tenacities of 70 000 to 80 000 lbs. per square inch and upward, and elastic limits rising from the lowest figures for Swedish irons and basic steels, perhaps 15 000 lbs., about a third the ultimate strength, up to 55 000 or 65 000 lbs., and two-thirds the ultimate tenacity, or more with hard steels.

Starting with the probably undeniable assumption that it is never desirable, in such work, to carry the loads above the elastic limits, under the most favorable circumstances, it is the necessary inference that the constants in the formula adopted must be made functions of the value of the elastic limit of the material selected for use. The formula must also adapt itself to the wide variety of constructive material now available, and actually in some degree made use of by the engineer. The general form of the expression being taken as here proposed,

$$S - a\,b\,R^2,$$

in which a is the stress to be allowed as a maximum, and b is the factor proportional to the assumed influence of the range-factor, R, the values of these quantities remain now to be adjusted to the quality of material to be used. The specified tenacity of the steel here considered as a typical quality seems low in consideration of the fact that the indications are, in some respects, very marked as promising the use of higher tenacities as improved manufacture gives better, more uniform and more resilient metals. Rail steel, for example, seems to be rising in standard, as also is steel for boiler plate, as the use of manganese is coming to be better understood.

The writer is very much inclined to expect that structural steel of all kinds will follow the course of rail steel, which has in the last few years changed from 0.25 or 0.35 carbon to 0.45 or 0.55; silicon from 0.05 up to 0.10; while the manganese has been, on the whole, rather reduced than otherwise by the improved materials and methods of steel making. In the common forms of steel, the relation of the elastic to the ultimate resistance is fairly constant at a ratio of six-tenths; but, should some of the later alloys come into use for structures, it may be expected that the proportion may raise to two-thirds. This will, presumably, make some difference in the margin of safety found desirable.

The particular fact to be noted in this connection is, that the elastic limit is the real gauge of the value of the material; whatever the nature or composition of the material, or whatever the character of the loads imposed. This being identified, the value of the member of whatever structure becomes settled. It is further to be noted, that the ratio of elastic limit to ultimate resistance and to safe load on the unit-area of section, for any given class of material, is substan-

tially constant for sound metal. It thus follows that, if it were possible to ascertain the elastic limit of any member or of any metal, the ultimate resistance would be computable, the safe load would become known, and the section suitable for a stated load, under specified conditions of loading, would become determinable. Mr. Thurston.

Ample experience and innumerable experiments have already shown that the loading of iron or steel to the elastic limit produces no permanent injury to the metal, and that the loading of any structure or of any member of a structure to the elastic limit, whether that limit is attained by static or dynamic loading, leaves the part practically as it was before loading. It simply takes out internal stresses up to that limit, and subsequent loads well within the limit do not affect either its method of yielding or its endurance.

All irons and steels employed in construction, when submitted to load of progressively increasing amount, up to and beyond the elastic limit, exhibit, on the stress-strain diagram representing the operation, a peculiar and unmistakable double curvature at the elastic limit, which exposes the location of that point with perfect accuracy, and the load sustainable there becomes constant, for a brief period, with continuing distortion. This marks, not only the elastic limit, but also the character of the material, and, if no other part of the diagram were obtained, the remainder of the curve could be easily and accurately laid down by anyone familiar with the material.

The testing of the piece up to the elastic limit thus serves quite as good a purpose as testing to destruction—with the decided advantage of saving the piece, leaving it as good as new and just as useful as a part of the proposed construction. This evidently applies to bridge rods or any parts in tension or compression, where the form and proportions of the piece are such as to permit the securing of a diagram such as is given by the ordinary test-piece. Placing it in test under loads approximating and passing the elastic limit, its ultimate and maximum carrying power becomes determined as completely as if it were actually broken. It can be then placed in the structure, and the engineer is as certain of its value and safety as if endowed with the gift of prophecy, as in fact he is in this instance. Every defect of composition, of make-up, or of molecular or mass structure, affects the elastic limit, and its rise above the normal intended indicates too great hardness and brittleness; its location at a lower figure than that proposed indicates either too soft a material or a faulty structure. If it has precisely the quality demanded by the specification, its elastic limit will be found at just the point anticipated, and the fact will be sufficient proof of compliance with the specification.

These principles were pointed out twenty years ago in a paper presented to this Society, March, 1878,* and illustrated by a set of

* *Transactions*, Am. Soc. C. E., Vol. vii, p. 53.

Mr. Thurston. figures which were practically representative of those obtained from the test of parts of the then famous Tariffville Bridge which had recently broken down with disastrous results.

In that instance, the test pieces from various members of the structure were found, after the accident, to have values indicated by the following figures, in which the loads are given in round numbers:

Resistances of Variously Strained Members.

	Elastic Limit.	Tenacity.
Sample No. 1............	16 500	46 000
" 2............	18 000	48 000
" 3	30 000	48 000
" 4............	22 500	50 000
" 5............	25 000	52 000
" 6............	27 500	52 000
" 7............	28 000	52 000
" 8............	30 000	52 000
" 9............	32 000	53 000
" 10............	34 000	53 000

The extensions were usually small, ranging in the neighborhood of 15%, sometimes falling to 10 per cent.

The figures show that the original quality of the iron was good, soft and of low tenacity, but fine. Its elastic limit and ultimate resistance, as shown by Nos. 1 and 2, were about 17 000 and 45 000, or 48 000 lbs. per square inch. The extension was about 20 per cent. The abnormal relation between the elastic limits and the tenacity in Nos. 3 and 6, and higher numbers, shows overstrain, raising the original to a new and abnormal limit, and measures the heaviest load carried on each piece during its life history.

The record shows that the highest elastic limits thus revealed are found on parts taken from the structure at points nearest the section of the bridge at which the break occurred, this being found, as would have been anticipated as probable, though not certain, in advance of the investigation. The maximum elastic limit of the body of the rod corresponds accurately to the maximum strength of the threaded part of the piece, and measures the load on the structure, at that point, at the instant of breaking. This load was found to be about double that of the actual weight of the load producing the disaster, and this indicates either a shock, as by derailment, or a flaw in the rod at that point, either fact giving a sufficient explanation of the accident.

"It would be concluded that the ordinary loads, such as had been carried previously to the entrance upon the bridge of that which caused its destruction, never exceeded, in their straining action, 16 500 lbs. per square inch of section of tension rod. * * * This accident was therefore caused by the entrance upon the bridge of a load capable of straining the metal to about one-half of its ultimate strength, if

slowly applied, but which, in consequence of its sudden application, doubled that stress. Mr. Thurston.

"This sudden action may have been a consequence, either of its coming upon the structure at a very high speed, or the result of the loosening of a nut, or of the breaking of a part of either the bridge floor or of one of the trucks of the train. The latter occurrence, permitting the load to fall even a very small distance, would be sufficient."

Thus the key to the character and resistances of the material and of the members of the structure is to be found in the measurement of the elastic limit, and of its relation to the ultimate resistance of the metal. From this somewhat extended discussion, one is led to the conclusion that the values of the constants, in the proposed formulas for constructions in iron and steel, should be based upon the elastic limits rather than upon the ultimate resistances of the material.

If this be granted, the constant a, instead of being taken at a conventional fraction of the ultimate load under static stress, would be properly taken at a rational proportion of the elastic limit of the class of material to be used, as unity for soft irons and mildest steels, and perhaps three-fourths or eight-tenths for the harder grades. In that case, the formula would stand as at present for irons, while its constants would increase with rising values of the strengths of the metals, but at a slower rate. For hard steels, for example, where the elastic limit is sometimes found as high as two-thirds the ultimate, the higher of these proportions would be desirable, and the actual limit figure would not be far from 25 tons, over 50 000 lbs., per square inch. Nickel steels, should they ultimately become available for structural purposes, and manganese steels of the higher grades, would give figures still higher, if no obstacle be found to an increase of their hardening elements such as has been found practicable in some other directions.

One very important fact is often overlooked in the study of formula construction for structures: a permanent load may effect a progressive destruction of over-strained materials of some classes, and, as the writer showed by experimental researches of many years ago, "a so-called factor of safety of 2 may prove to be, after a time, a factor of unity." In other words, the indications of those experiments were that a factor of 2, based upon ultimate resistance, or 1 as based upon the elastic limit, might not certainly prevent fracture under static load, if it were sufficiently long imposed. Thus, taking 2 as the minimum limit of the factor for absolute dead loads, and 2 for the factor of ordinary risks, including what Holley called the factor of ignorance, it would seem that the product, 4, is a minimum for the most favorable cases, even where no live load is carried at all. This limiting value of the factor to be adopted must finally be increased in the proportion indicated by the researches giving as their outcome the formulas which have been discussed, and of which it would seem that the range-formula gives most satisfactory results as to form.

Mr. Thurston. In thus attempting to settle upon a factor of safety, upon either basis, with various qualities of material, such as are now available and actually more or less used in engineering, there always remains the uncertainty which is known to arise, even with the same material, as a result of known or unknown differences in the physical condition produced by differences in mechanical treatment. This is illustrated by the following experiment:

In the year 1883, the writer suspended a number of iron wires, under carefully adjusted and attached static loads, ranging, by differences of 5%, from 95% to 65% of their ascertained breaking loads under the ordinary tests, where the testing machine produces prompt rupture by a steadily augmented tension, unintermitted to the point of rupture. Two qualities of the same material were used; the one being set up hard-drawn as it came from the wire block, the other having been carefully annealed after the last reduction. The results of this experiment were as given in Table No. 11.

TABLE No. 11.—ENDURANCE OF IRON UNDER DEAD LOADS.

Per cent. maximum static load.	Time Under Load Before Fracture.	
	Hard, unannealed wire.	Soft, annealed wire.
95	8 days	3 minutes.
90	35 days	5 minutes.
85	Unnoted, but one or two years	261 days.
80	91 days	266 days.
75	Unbroken after several years	17 days.
70	Same results	455 days.
65	Same results	455 days (probable jar).

Some of these wires were still unbroken in 1898, after fifteen years' loading; but, some slight oxidation setting in, the further prosecution of the experiment promises less certain deductions.

The significance of this research and its results becomes particularly notable when it is remembered that probably no two bars come from the rolls in the same state of hardening by mechanical strain or of annealing by the relative rates of heating and of cooling, in the process of production and subsequent treatment. The differences are probably never as great as those observed in the case above described, but they are of the same nature and may sometimes be considerable. In the case just related, the safety of the hard iron was evidently assured by a factor of safety of 2; while it is just as evident that a factor of 2 for the soft iron is, in fact, not certainly a factor of absolute safety at all. It still remains for special research to discover what bearing these facts have upon the safe proportions

of bridge-members and other constructions. At present it can only be said that great caution must be observed in the choice of material and in prescribing its treatment, as well as in the use of constants for the formula to be employed where computing for repeated and reversed loading. Mr. Thurston.

The writer would be inclined to adopt not over 5 tons, or, say, 12 000 lbs. per square inch for an average and good quality of what is recognized as the best bridge-irons, and would prefer them hard-rolled. For steels, perhaps one-half of the figure found for the elastic limit would serve for the value of the constant a until some of these still misty conditions and effects are more completely cleared up by further research. The writer would like to see the idea put in practice of testing construction material to the point of renewed elevation of the resistance under strain, beyond the elastic limit, to determine whether it shall be introduced into the structure. It would certainly, in the opinion of some of those who have carefully followed the investigation above related, prove a very satisfactory method of securing for the designer and constructor some confidence, if not certainty, that such members are actually capable of carrying fully the amount of load assigned them. Under the usual system, of testing by sample, the engineer always feels that it is extremely probable that he has tested sound specimens, while there have still more probably gone into the structure the exceptionally unsound and defective pieces received in the same lot. The discussion of these points by the writer, in earlier volumes of the *Transactions* and elsewhere, has furnished additions to the record which may, perhaps, prove of considerable interest in this connection.* Meantime the paper under discussion will certainly stand in those records, permanently, as a valuable and exceedingly complete and available presentation of the subject, up to its date.

J. A. L. Waddell, M. Am. Soc. C. E.—The subject relates to one of the most important unsolved problems in the engineering profession, because, unless steel for all bridge members of all spans is strained about right, either metal is wasted or the ultimate danger limit is encroached upon in making bridge designs. The principal points upon which knowledge is necessary are the extent to which the metal in the various members of all approved types of modern bridges is strained by moving loads applied at different velocities, and the relation of the effects of loads so applied to the effects of the same loads applied statically. Mr. Waddell.

If such data were obtained, there could readily be constructed a diagram of percentages to add to live load stresses that are determined on the assumption of static application of loading, which would cover

* See Thurston's "Materials of Engineering," Vol. ii, Chap. x, and, particularly, Sec. 293 to 303, inclusive; also *Transactions*, Am. Soc. C. E., 1874, 1876, 1877, 1878, 1880, where are discussed the effects of time and load.

Mr. Waddell. the combined effects of impact, shock, jar, vibration, etc., all of which factors tend to increase the actual intensities of working stresses. The addition of these percentages would reduce the total stresses to equivalent static stresses, and would permit the adoption of one or two unit working stresses of each kind for designing. An allowance for unavoidable, small secondary stresses could either be included in this percentage diagram, or else this feature could be taken care of in determining the intensities of working stresses to use with equivalent static stresses.

The results of deflection observations on Indian bridges, quoted by the author, are certainly both interesting and valuable, but it is doubtful whether they will apply satisfactorily to modern American bridges, because the types of structures used in the two countries differ widely. Speaking generally, the weight of metal in an Indian bridge exceeds that in a corresponding American bridge, of the same span and loading, by from 60 to 100 per cent. This does not mean that the Indian bridge is so much stronger, stiffer, or better, but that the extra metal is simply wasted, its only good function being to absorb shock and impact. Such a result can be obtained just as well by using a material less expensive than steel, for instance, stone ballast in the floor. Again, Indian bridges are much more shallow than American bridges, and have far shorter panels and many more parts.

The writer has reason to think that the Indian experiments were made mainly upon spans as a whole, and not upon their component members; consequently, the results will apply only to chords as a whole and not to webs, for the writer does not agree with the author in the statement that "the results obtained from the deflection of the truss, as a whole, must to a great extent represent the average for all of its members."

Not only are hangers and other light web members much more subject to shock than are heavy chords, but also the light end panels of a bottom chord are probably somewhat more affected by impact, etc., than are the heavier chord members at and near mid-span. Experiment alone will determine the truth of this, and settle the vexed question of what are the various actual intensities of stress caused by live loads applied dynamically. The writer has recently expressed himself fully concerning this question of impact.*

The writer is of the opinion that a series of tests on actual intensities of working stresses for bridge members should be made. It would probably involve an outlay of from $100 000 to $200 000 to make these tests properly. An advisory committee of, say, five prominent bridge engineers should be retained to lay out, formulate and systematize the work, and to direct a working committee of three well-paid engineers who would devote their entire time and energies

* "De Pontibus."

to the investigation. Two members of the latter committee should be experienced experimenters and the other an expert bridge engineer. Mr. Waddell.

Most of the experimenting should be confined to modern structures of the most approved type; but, if time and funds would permit, some work should be done on bridges of the older types, many of which are still in common use. The beneficial results of such a series of tests are so apparent that it would be useless to discuss them.

The writer is surprised to see how large a portion of the paper is devoted to iron bridges, which are now things of the past, and how small a portion to steel bridges. The benefits to be derived from impact investigations are confined almost entirely to future bridges, which certainly will be of steel; because the practical railway engineer, in determining whether it is safe or not to continue to use an existing bridge, bases his judgment mainly upon the action of the structure under load, the excellence or weakness of its details, and its location in respect to traffic and shock, as well as upon the intensities of working stresses to which it is subjected.

It is questionable whether the author's method of tabulating or plotting his percentages of increase for live load stresses upon the basis of ratio of dead and live load stresses, is as correct or scientific as the more usual manner of basing them upon length of span covered by moving load; certainly it is by no means as convenient for the computor, who does not care to spend any unnecessary time in figuring ratios of loads, when he can find the percentage by simply glancing at a diagram, after noting the length of span to be covered for the greatest stress on the piece considered. In respect to the correctness of the author's method, let an extreme case be assumed for comparison, viz., a 200-ft. span of ordinary type, and a 100-ft. span so overloaded with ballast floor as to make the ratio of dead and live loads the same for the two spans. Would it be better to use the same percentage of increase for these two bridges or to make it greater for the short span than for the long one? Most engineers would certainly adopt the latter method.

The writer has never yet been convinced of the applicability of the results of the investigations conducted by German scientists on fatigue of metals to the proportioning of bridges. In order to obtain any results at all, the metal had to be strained far beyond the elastic limit, and the loads had to be applied every few seconds, while in bridges the metal never is (or, more strictly speaking, never should be) strained much higher than about one-half of the elastic limit, and the loads are applied at much longer intervals, thus giving the metal a chance to recover itself and return to its original state before the application of another load.

The determination of the value or values of intensities of working

Mr. Waddell. stresses is purely a matter of professional judgment, based upon experience in the field, and should not be relegated to scientists in their laboratories.

If the elastic limit is recognized as the true criterion of ultimate strength, and if the strain is no higher than one-half of this amount for equivalent static stresses due to ordinary loads, and 30% is added for the same, due to an unusual or practically impossible combination of all loads, including wind pressure, a sufficient margin will have been allowed to cover small secondary stresses, possible flaws in the metal and occasional dropping below average in elastic limit.

For medium steel having an ultimate tensile strength (measured on specimens) between 60 000 and 70 000 lbs. per square inch, and a least elastic limit of 35 000 lbs. per square inch, the writer in his latest specifications strains eye-bars 18 000 lbs. per square inch and built members in tension 16 000 lbs. per square inch.

The author evidently has more faith in metal than has the writer, for the former strains mild steel, having an average ultimate strength of 60 500 lbs. (27 English tons) per square inch, 12% higher than the writer does the medium steel. It is true that the author counts upon 40 300 lbs. (18 English tons) per square inch as the elastic limit of such mild steel; but the writer's experience in the use of metal does not indicate that it is safe to rely upon more than 33 000 lbs. per square inch as the elastic limit of such metal, to say nothing of the probable decrease in elastic limit between full-size members and test specimens. Standard specifications for soft steel allow the elastic limit to run generally as low as 30 000 lbs. per square inch, and sometimes even down to 27 000 lbs.

Referring to the Table No. 9, as a matter of curiosity, the writer has made a comparison of the nominal stresses involved, using the impact formula and intensities of *De Pontibus*, with those given by the author, using as a basis the bottom chords of single-track railroad bridges.

The results of the comparison are given in Table No. 12.

TABLE No. 12.

Span in feet.	Live load.	Impact.	Dead load.	Nominal intensity.	Stone's nominal intensity.	Percentage of difference.
100.........	4 366	3 844	1 630	12 144	14 280	+17.6
150.........	4 144	3 550	1 850	12 621	14 806	+17.3
200.........	4 030	3 303	2 130	13 102	15 366	+17.3
250.........	3 930	2 096	2 444	13 546	15 895	+17.3
300.........	3 860	1 930	2 820	13 966	16 484	+17.6
350.........	3 800	1 788	3 165	14 322	16 823	+17.5
400.........	3 760	1 671	3 500	14 631	17 150	+17.1
450.........	3 730	1 571	3 830	14 914	17 450	+17.0
500.........	3 700	1 4[illegible]0	4 226	15 169	17 713	+16.8
550.........	3 680	1 402	4 576	15 389	17 932	+16.5
600.........	3 660	1 331	4 930	15 585	18 120	+16.3
				Average difference............		+17.1

It will be noticed that the author strains his mild steel nominally about 17% higher than the writer strains his medium steel, and that the variations in the differences are small. Mr. Waddell.

J. L. POWER O'HANLY, M. Am. Soc. C. E.—The deflection of a beam is the effect of an applied vertical force. Horizontal stress will not cause an increase of deflection. Hence, whenever a deflection is observed in a bridge, in excess of that due to the weight of the structure and its external quiescent load, the excess must be due to impact and acceleration, practically to impact. No other possible cause can be assigned for the phenomenon. For, were track and machinery perfect, the deflection of the bridge would be that due to the static load only, irrespective of speed of train. Hence, the observed difference is due to the imperfections of track and machinery. Increase of deflection may, therefore, be defined as the measure of the intensity of the impact. Mr. O'Hanly.

The paper is a useful addition to the existing scant literature on this very important subject, and will be hailed as of great assistance in solving the disputed question of the percentage of stress due to impact.

The writer believes that in designing railroad bridges of ordinary spans, the whole load, live and dead, should, in calculating stresses, be treated as all live load with a coefficient of 2.0. It is a safe rule, and takes some cognizance of the wants of the not distant future. In long-span bridges, however, with the dead load per linear unit clearly in excess of the train load, the line of demarcation between dead and live load may profitably be drawn, and a practice like that so well advôcated by the author safely and economically introduced.

There can be little doubt that repeated applications of a load, particularly as it approaches the elastic limit of the material, impairs the strength, stiffness and efficiency of a bar and hastens its rupture. Universal experience confirms this doctrine, and brings the proposition within the definition of an axiom. To Wöhler is the great merit of bringing the problem within definitely assignable limits. The choice, however, of the expression fatigue, by scientists, to convey the idea, has been most unfortunate, almost bringing a sound and rational theory into ridicule, if not contempt. Fatigue will ever be associated with, and exclusively applied to, the effect of strain on animal tissue or living organism; nor is there any good reason to suppose that the feeling or sensation awakened thereby is aught akin to the severing or relative displacement of the molecules of iron or other inanimate substances.

In his examination of the tables, the writer is obliged to confess, even at the cost of unmasking his own ignorance, that he is unable, in many instances, to verify the author's figures.

Table No. 1.—It is to be regretted that the author did not find it convenient to accompany the paper with the data on which this table

Mr. O'Hanly. is founded, or at least as much thereof as would make it more comprehensible to those less fortunately circumstanced. The writer assumes that the results in the third column are based on the recorded deflection experiments, but how arrived at he is unable to surmise, except from the author's rather meager account.

Contrary to expectation, the writer finds that the results in column 4 are not strictly proportional to the corresponding figures in column 3, that is to say, the square inch ratio of

$$\frac{\text{Static breaking stress}}{\text{Compound load breaking stress}} < \frac{\text{Immediate percentage effect}}{\text{Live load percentage}}.$$

$$\text{For example, } \frac{21.00}{14.36} < \frac{147.4}{97.5},\ 1.46 < 1.51.$$

It is also less than the ratio of whole load. Thus:

$$\frac{21.00}{14.36} < \frac{147.4}{100},\ \text{or } 1.462 < 1.474.$$

The table gives the percentage of immediate effect of live compared with dead load, the breaking stress per square inch of section corresponding to such percentage of dead to live load, and the relative proportions of such stress per square inch due to dead and live loads respectively. Take, for example, the case of 50% dead and live loads. The breaking stress per square inch for this percentage is 20.60 tons, equally apportioned—10.30 tons—among the dead and live load components. As the writer understands the table, the immediate effect of 50 tons of live and dead loads respectively is equivalent to 101.94 tons of quiescent load, and it is surmised that the relative breaking stresses of the different kinds of load would be in like proportion, that is to say, 101.94 : 50 :: 20.6 : 10.1 breaking stress per square inch, due to dead load, and 10.5 tons to live, as against 10.3 in the table. If the exception be well taken, like reasoning applies to all the figures of columns 5 and 6.

Table No. 2.—The writer is unable to verify the figures in column 3 of this table, but fancies he has solved those of column 6, with the the aid of Gerber's formula—(Min. $S + \frac{1}{2}\triangle)^2 + 28\triangle = t^2$—as given by the author, from which the figures of columns 4 and 5 are readily deducible. Example—percentages of live and dead load respectively, 99 and 1. Then :

$$99 : 1 :: 14 : .14 = \text{Min. } S.$$
$$(0.14 + \tfrac{1}{2}\triangle)^2 + 28\triangle = 21^2.$$
$$0.02 + 0.14\triangle + \tfrac{1}{4}\triangle^2 + 28\triangle = 441.$$
$$\tfrac{1}{4}\triangle^2 + 28.14\triangle = 441 - 0.02 = 440.98.$$
$$\triangle^2 + 112.56\triangle = 1\,763.92.$$
$$\triangle^2 + 112.56\triangle + 56.28^2 = 1\,763.92 + 3\,167.44 = 4\,931.36.$$
$$\triangle + 56.28 = 70.22.\quad \triangle = 70.22 - 56.28 = 13.94.$$

TABLE No. 13.

Mr. O'Hanly.

Load Percentages.		Breaking stress due to live load.	Tabular numbers.
Dead.	Live.		
0	100	14.00	14.00
1	99	13.94	......
2	98	13.89	......
2.5	97.5	13.86	13.86
3	97	13.82	......
4	96	13.76	......
5	95	13.70	13.70
6	94	13.65	......
7	93	13.58	......
8	92	13.50	......
9	91	13.43	......
10	90	13.38	13.38
11	89	13.26	......
12	88	13.22	......
13	87	13.14	......
14	86	13.08	......
15	85	13.06	13.06
16	84	12.98	......
17	83	12.90	......
18	82	12.82	......
19	81	12.65	......
20	80	12.57	12.57
21	79	12.46	......
22	78	12.39	......
23	77	12.28	......
24	76	12.19	......
25	75	12.11	12.11
26	74	12.00	......
27	73	11.89	......
28	72	11.80	......
29	71	11.69	......
30	70	11.60	11.60
31	69	11.47	......
32	68	11.35	......
33	67	11.25	......
33⅓	66⅔	11.23	11.24
34	66	11.15	......
35	65	11.02	11.05
36	64	10.90	......
37	63	10.79	......
38	62	10.66	......
39	61	10.55	......
40	60	10.45	10.45

Columns 4 and 5 are easily deducible from column 6.

This table gives the percentages of cumulative effect of live compared with dead load, the breaking stress per square inch of section corresponding to such percentages of dead to live load, and the relative proportions of such stresses per square inch due to dead and live loads, respectively. Take, for example, the case of 50% dead and live loads. The breaking stress per square inch for this percentage is 18.20 tons, apportioned equally among the dead and live load components. The cumulative effect of 50 tons live and dead loads respectively is equivalent to 115.41 tons of quiescent load, the 50 tons of live load being equivalent to 65.41 tons ; and it is surmised that the relative breaking stresses per square inch of the different kinds of load

Mr. O'Hanly. are in like proportion. That is to say, 115.41 : 50 : : 18.20 : 7.88 tons breaking stress per square inch due to dead load, and 10.32 to live, as against 9.10 tons in the table. This reasoning, if correct, equally applies to the figures of columns 5 and 6.

Table No. 3.—The writer's mental darkness as to column 3 of this table, if not more intense, is just as persistent. With the aid, however, of Launhardt's formula, $a = 14 \left(1 + \frac{\varphi}{2}\right)$, as given by the author, he has been enabled to verify the figures in column 4 from which those of columns 5 and 6 are readily deducible.

The table gives the percentages of cumulative effect of live load compared with dead load, the breaking stress per square inch of section corresponding to such percentages of dead to live load, and the relative proportions of such stresses per square inch due to live and dead loads, respectively. Take, for example, the case of 50% dead and live loads. The breaking stress per square inch for this percentage is 17.43 tons, equally apportioned in the table between the dead and live components. The cumulative effect of 50 tons live and dead loads, respectively, is equivalent to 120.45 tons of quiescent load; and it is surmised that the relative breaking stresses per square inch of the different kinds of load are in like proportion. That is to say, 120.45 : 50 : : 17.43 : 7.23 tons breaking stress per square inch due to dead, and 10.20 to live, as against 8.72 tons in table. The reasoning applies equally to all the figures of columns 5 and 6.

Table No. 5.—(*b*) Bell-Robertson Rule.

(1) As the writer understands this rule, which is given on page 491, it implies that in order to obtain the stress on the bridge equivalent to all static load, the whole load is to be multiplied by 1.5, and to the product the dead load added for the equivalent compound load, provided that the dead load is not less than half the live load. But should it be less than one-half, then half the live load shall be substituted therefor.

The tabular number corresponding to 2.5% dead and 97.5 live is 197.44. Applying the rule, the result becomes

$$100 \times 1.5 + \frac{97.5}{2} = 198.75,$$

as against 197.44 in Table No. 14.

(2) Construing the rule to imply that only the live load is to be multiplied by 1.5, and the dead load added to the product when not less than half the live, the result of the above would be

$$97.5 \times 1.5 + \frac{97.5}{2} = 195.0,$$

as against 197.44 in Table No. 14.

TABLE No. 14.

Mr. O'Hanly.

PERCENTAGE OF LOAD.		Coefficient percentage for whole load.	Coefficient percentage for moving load only.	Tabular number.
Dead.	Live.			
2.5	97.5	198.75	195.00	197.44
5	95	197.50	190.00	194.74
10	90	195.00	180.00	188.89
15	85	192.50	170.00	182.35
20	80	190.00	160.00	175.00
25	75	187.50	150.00	170.70
30	70	185.00	140.00	166.67
33⅓	66⅔	183.33	133.33	150.00

It appears that a mean of the two comes nearer the tabular number than either.

(c) Range Coefficient Rule.—The writer's calculations agree with the figures in the table.

(d) Launhardt's Rule.—The author says:

"The use of this formula, with the values adopted, involves the following assumptions:

"(a) The coefficient to be applied to the Moving Load, to obtain its equivalent in terms of Fixed Load, is 2.0 for the condition of 'All Moving Load,' diminishing as the ratio of Moving Load to Fixed Load decreases, until it becomes 1.5 for the condition of 'All Fixed Load.' The variation in the value of the coefficient is directly in the ratio of Total Load to Fixed plus Total Load."

1. As the writer understands this rule, it implies that the coefficient 2 for all live load shall diminish in the ratio of total load to total load plus fixed load. That is to say, if the total load were 100 tons, of which the dead part 10, then 110 : 100 : : 200 : 181.81. If to this be added the dead load, the coefficient percentage becomes 191.81, with the tabular number 190.91. A table computed on this assumption gives the following results:

TABLE No. 15.

LOAD PERCENTAGE.		Coefficient percentage.	Coefficient percentage with dead load added.	Tabular number.
Dead.	Live.			
2.5	97.5	195.12	197.62	197.56
5	95	190.48	195.48	195.24
10	90	181.82	191.82	190.91
15	85	173.91	188.91	186.96
20	80	166.67	186.67	183.33
25	75	160.00	185.00	180.00
30	70	153.85	183.85	176.92
33⅓	66⅔	150.00	183.33	175.00

Mr. O'Hanly. 2. Assuming that the coefficient shall vary with each succeeding coefficient, the following result is obtained.

For example, suppose dead and live loads to be, respectively, 2.5 and 97.5 percentages. Then 102.5 : 100 : : 200 : 195.12. If to this be added the percentage of dead load, the sum gives 197.62. The corresponding tabular number is 197.56.

TABLE No. 16.

Load Percentage.		Coefficient percentage.	Coefficient percentage with dead load added.	Tabular number.
Dead.	Live.			
2.5	97.5	195.12	197.62	197.56
5	95	192.80	195.30	195.24
10	90	186.00	191.00	190.91
15	85	181.90	186.90	186.96
20	80	178.00	183.00	183.33
25	75	174.29	179.29	180.00
30	70	170.75	175.75	175.92
33⅓	66⅔	170.08	173.41	175.00
75	25	151.26	156.26	157.14
90	10	146.72	151.72	152.63

3. A closer approximation is obtained by assuming the percentages to vary by units. Thus, 101 : 100 : : 200 : 198.02. To which adding the dead load gives 199.02. Table No. 17 is constructed on this basis.

TABLE No. 17.

Load Percentages.		Coefficient percentage.	Coefficient percentage with dead load added.	Tabular number.
Dead.	Live.			
1	99	198.02	199.02	......
2	98	197.05	198.05	......
3	97	196.09	197.09	......
4	96	195.14	196.14	......
5	95	194.17	195.17	195.24
6	94	193.24	194.24	......
7	93	192.32	193.32	......
8	92	191.46	192.46	......
9	91	190.55	191.55	......
10	90	189.65	190.65	190.91
11	89	188.76	189.76	......
12	88	187.88	188.88	......
13	87	187.01	188.01	......
14	86	186.15	187.15	......
15	85	185.30	186.30	186.96
16	84	184.45	185.45	......
17	83	183.61	184.61	......
18	82	182.78	183.78	......
19	81	181.97	182.97	......
20	80	181.16	182.16	183.33

(e) Range Formula.—The writer has signally failed in all his attempts to verify the author's figures, as the following exhibit shows. Mr. O'Hanly.

All Moving Load.—$a = 21 - 12\ R^2 = 21 - 12 = 9$.

Then, $9 : 21 : : 100 : 233.33$

per cent. coefficient for all live load.

Example: Dead load 2.5%, live 97.5.

$$R = \frac{97.5}{100} = 0.975.\ R^2 = 0.95,\ \therefore$$

$$21 - 12\ R^2 = 21 - 11.4 = 9.6.$$

Then, $9.6 : 21 : : 100 : 218.75$.

Tabular number, 221.96.

TABLE No. 18.

Load Percentage.		Coefficient percentage.	Coefficient percentage with dead load added.	Tabular number.
Dead.	Live.			
2.5	97.5	218.75	221.25	221.96
5	95	206.49	211.49	212.09
10	90	186.17	196.17	195.74
15	85	170.32	185.32	182.73
20	80	157.66	177.66	172.07
25	75	147.37	172.37	163.16
30	70	138.95	168.95	155.56
33⅓	66⅔	134.04	167.37	133.33

Table No. 6.—The writer finds the figures in this table agree with his calculations, as far as verified.

Table No. 7.—The writer finds the figures of this table, as far as verified, agree with his calculations, taking the ratio of safe stress to experimental in Table No. 6.

Table No. 8.—This is merely a compilation in a single table of the results of Tables Nos. 5, 6 and 7 for the "Range Formula," the author's favorite rule.

Table No. 9.—This table is for steel structures what Table 8 is for wrought iron. The writer in this case encounters the same insuperable difficulty as for wrought iron, in finding the coefficient for steel to be multiplied into live load for reducing it to equivalent dead load.

The author well deserves the thanks of the profession for the thought and labor involved in the preparation of an interesting paper on a most important subject.

F. E. Turneaure, Assoc. Am. Soc. C. E.—In discussing the subject of live loads, the author makes certain statements which seem to the writer to be open to criticism. Mr. Turneaure

Mention is made of the live load of the text books as meaning a load suddenly applied in a vertical direction, and, therefore, not corresponding to the moving load of a bridge. Certainly no such notion

Mr. Turneaure. of live load is conveyed by text books with which the writer is familiar, the term "live load" being used therein to indicate a moving load, and in no way connected with the suddenly applied load mentioned above.

The statement is also made that, neglecting the effects of vibration and deflection, the faster an engine runs in a horizontal direction the less its vertical effect. As the writer sees it, if the motion is strictly horizontal there is no vertical component, and, therefore, there can be no unbalanced vertical force acting on the body. This being the case, it follows that the upward pressure of the bridge on the moving body is at all times equal to the force of gravity acting downward, or, in other words, is constant for all velocities. If the deflection of the bridge causes the body to move in a path curved below the horizontal, there will be added to the effect of gravity that due to centrifugal force, which increases with increase in speed.

The author has done the profession a great service in separating and discussing so thoroughly the two effects of the moving load. These effects are too often treated apparently as one, but the distinction between them needs to be kept clearly in mind in discussing the subject of working formulas. It appears to the writer that the simplest way to treat the two effects is to add a percentage to the computed live-load stresses for impact, or immediate effect, and then to treat the stresses so found as the correct maximum live load stresses to be used in formulas taking account of fatigue, or cumulative effect. Practically the same results can, no doubt, be obtained in any given series of designs by the use of a single formula, as the author shows, but the use of a separate impact formula seems to be a more scientific procedure, and one that better enables the effect of each element to be appreciated and the judgment to be brought to bear on the selection of constants. Such a formula can also be more readily modified as our knowledge of impact increases.

The results of tests concerning the effects of live loads are very important, and it would be desirable to know more in detail of the tests whose results are summarized in the paper. A very important point is that of speed of trains used in the tests. It is to be presumed that the ratios plotted in Fig. 1 represent the total effect of the live load, including that of vibration, as compared with the effect of the same load moving at a very slow speed or standing still on the structure; but the writer would like to be assured on this point. It seems quite desirable to know, also, how far the maximum ratios exceed the averages plotted, as it is the maximum and not the average that must be provided for. The ratios given by the author are very much less than those observed by Professor Robinson*, and likewise much less than those found by the writer in a series of tests recently made on a

* "Vibration of Bridges," S. W. Robinson, M. Am. Soc. C. E. *Transactions*, Am. Soc. C. E., Vol. xvi, 1887, p. 42.

number of bridges. The results of these tests agree in a general way with those made by Professor Robinson, and indicate that, for American practice at least, the percentages to be added for impact should be two or three times as great, for spans from 100 ft. to 200 ft. in length, as those given by the average curve in Fig. 1. Mr. Turneaure.

WILLIAM CAIN, M. Am. Soc. C. E.—The author has made a valuable contribution to the study of permissible unit stresses in the members of a bridge truss. From about 1 500 separate observations on deflections of bridges in India, the "immediate effect" of the moving load is obtained, it being assumed that the stress in any member is proportional to the deflection of the bridge—presumably at the center. Thus, for a fixed load, A tons per lineal foot and a moving load, B tons per lineal foot, Table No. 1, gives the factor by which B must be multiplied to give B', the corresponding static load that will cause the same deflection as B in motion. Mr. Cain.

There are doubtless additional stresses caused by impact and the resulting vibration (causing the rattling of the smaller members) that do not show in deflection, but these are neglected. For those engineers who do not believe that repetition of stress below the elastic limit causes any fatigue of metal, the cross-section of a member in square inches is logically found by dividing the stress due, not to the load $(A + B)$, but $(A + B')$, by 7, the safe unit stress allowed by the author, in tons per square inch.

The author agrees with many engineers in considering that millions of repetitions of stress should be allowed for by Gerber's or Launhardt's formula. To take the simpler of the two, or Launhardt's formula, the method now to follow to find the cross section of a lower chord member say, is first to find the stress in it corresponding to the load $(A + B')$ above and divide it by

$$\frac{14}{3}\left(1 + \frac{1}{2}\,\frac{A}{A + B'}\right).$$

Table No. 3 gives the factor f by which B' must be multiplied, so that the stress corresponding to $(A + f\,B')$ when divided by 7 shall give the same cross-section as that just found. In fact, since the stress is proportional to the load per foot, the value of f can be found from

$$\frac{A + B'}{\frac{14}{3}\left(1 + \frac{1}{2}\,\frac{A}{A + B'}\right)} = \frac{A + f\,B'}{7}.$$

Also, under "method of calculation" (page 481) is given the process of finding the coefficients in Table No. 4, so that B multiplied by the proper coefficient in this table shall equal $f\,B'$. This product added to A gives, thus, the load per foot to be treated as fixed load in finding the stress in the lower chord member supposed. On dividing

Mr. Cain. this stress by 7, the same area of cross-section is found as before. Thus, by use of Table No. 4, the stress resulting is simply divided by 7, which is simpler than the preceding method involving the direct use of Launhardt's formula.

The solution is correct for the chords, but it is to be inferred that the author supposes the same coefficients to apply to the web, which is erroneous. This would mean, in fact, that for a given fixed and moving load, the same unit stress is to be used throughout for tension in lower chord and every web member. The unit stress in a web member at the center, which varies from zero to its maximum, is thus to be taken the same as for the lower chord, which is inconsistent with Launhardt's formula.

To show where the error is introduced, it is to be first noted that the assumption on which Table No. 1 is founded is that the stress in any member of a bridge truss (leaving out the counters) is directly proportional to deflection. This may be granted.

The load, A fixed and B moving, is thus equivalent, by Table No. 1, to load $(A + B')$ fixed, and the stress can be found for this equivalent static load; but to get the cross-section of any web-member this stress must be divided by

$$\frac{14}{3}\left(1 + \frac{1}{2}\varphi\right),$$

where φ is not equal to $\frac{A}{A + B'}$ as for the chords, but to the actual minimum stress experienced in the passage of the moving load on the web-member divided by the maximum stress. But $\varphi =$ Min. $S \div$ Max. S for a web member, varies from zero at the center of the truss to a value about equal that for the chords at the ends, and it is thus plainly inadmissible to take it the same throughout as for the chords.

It is equally inaccurate to take the true value of φ for a web member in applying Table No. 1, as the ratio of fixed to total load covering the entire bridge is used as the argument in the table, and this does not correspond to Min. $S \div$ Max. S for the web member. Besides Min. S in a web member corresponds to a different position of the moving load from that pertaining to Max S, whereas Table No. 1 refers to the same position. If a choice has to be made in order to utilize Table No. 4, then it is plainly best to use the latter method and replace the ratio of fixed to fixed and moving load in Table No. 4 by Min. $S \div$ Max. S for the web-member considered, for this gives the least cross-sections. This is the plan adopted in America, where the modified Launhardt formula is used for unit stresses.

It would seem, therefore, that the tables following Table No. 1 apply strictly only to the chords. For such members they are extremely interesting and instructive. Referring to Fig. 3, it will be observed that the straight line marked "Launhardt's system," is

much nearer the curve given by experiment than the one correspond- Mr. Cain.
ing to Cooper's "fixed coefficient," and on account of its simplicity it is to be commended.

The author's formula, $a = 21 - 12 R^2$, is supposed to allow, too, for increased stresses due to vibration, but there can be but little vibration in the chords, so that, although the author's formula is somewhat nearer the results of experiment than the modified Launhardt, still the simplicity of the latter appeals to the writer, and in the present state of knowledge respecting the effect of repetition of stress within the elastic limit, he sees no reason for adopting any more complicated formula, particularly noting what was said with respect to the additional approximations pertaining to web-members.

The formula for unit stress in pounds per square inch for wrought-iron ties, $a = 7\,500\ (1 + \varphi)$, was first proposed in *Van Nostrand's Magazine*,* the term $\frac{1}{2}\ \varphi$ of Launhardt's formula being empirically changed to φ to allow for all the influences of impact. The paper by J. M. Wilson, M. Am. Soc. C. E.,† shows that it was adopted with notable additions, before 1886, on the Pennsylvania Railroad, and it soon came into more general use. Later, the attempt was made by the writer, in view of Professor Robinson's experiments on deflection of American bridges,‡ to ascertain more nearly the scientific basis of the modification, and the results of the investigation were given in *Engineering News*,§ but the cream of the whole matter will be found in the discussion of the paper by J. A. L. Waddell, M. Am. Soc. C. E., entitled "Some Disputed Points in Railway Bridge Designing."¶ Some of the leading points from that paper will now be given in a very brief form.

Launhardt's formula for breaking unit stress for millions of repetitions can be written,

$$\text{Unit stress} = \text{constant}\left(1 + \frac{1}{2}\ \varphi\right).$$

For a less number of repetitions, it is found, by working over the values given in the original experiments, that the coefficient $\frac{1}{2}$ is decreased and becomes quite small for a few hundred thousand repetitions, which is nearer the number to be provided for in most bridges than the former.

Taking A = stress in a lower chord member due to weight of bridge, B = stress due to live load at rest and Bp = superadded stress corresponding to the extra deflection due to the train's motion, then the actual stress in the member as the train rolls over is $A + B + Bp$.

* November, 1877, p. 459.

† "On Specifications for the Strength of Iron Bridges," *Transactions*, Am. Soc. C. E., Vol. xv, p. 389.

‡ "Vibration of Bridges," by S. W. Robinson, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. xvi, p. 42.

§ July 16th, 1887, *et seq*.

¶ *Transactions*, Am. Soc. C. E., Vol. xxvi, p. 208.

Mr. Cain. Doubtless the coefficient φ in the Launhardt formula is less than $\frac{1}{2}$ for the average bridge, particularly where safe stress is considered below the elastic limit, but assume it at $\frac{1}{2}$ as a maximum, to account not only for repetition of stress, but also for the additional stresses caused by tremor not giving any vertical deflection, and write the safe stress in pounds per square inch,

$$b = 10\,000\left(1 + \frac{1}{2}\varphi\right),$$

whence the cross-section of the lower chord member will be,

$$\frac{A + B(1 + p)}{10\,000\left(1 + \frac{1}{2}\varphi\right)}.$$

Now if this same cross-section is found by dividing $A + B$ simply by the unit stress, $a = 7\,500\,(1 + \varphi)$, giving,

$$\frac{A + B}{7\,500\,(1 + \varphi)},$$

and this expression is placed equal to the other one, the following values of p are obtained:

Span in feet....	20	70	150	300	450
φ.............	0.10	0.20	0.33	0.50	0.66
p.............	0.30	0.28	0.25	0.22	0.20.

The values of φ given here are for old bridges, but as they sufficiently show the variations of p for modern bridges, it was not thought worth while to make the computations over again.

Professor Robinson's observations on deflection, previously referred to, show that the increased live load deflection due to the motion of freight and passenger trains at usual speeds varied all the way from almost nothing to 28% of the deflection caused by the same trains at rest.

In place of taking the average, it is certainly safer to take the mean of 25, giving the largest deflections, out of 193 transits observed, or 18.4%. This is considerably less than the maximum 28.

The bridges were open web bridges, 13 in all, of spans varying from 128 to 189 ft.

Thousands of observations on deflections of American bridges are needed to correspond in accuracy of results with the author's Indian experiments, but it is seen from the table above that the use of the formula, $(A + B) \div 7\,500\,(1 + \varphi)$, entails a value of p about 25% for the kinds of spans examined.

The experiments in India gave for $\varphi = \frac{1}{3}$ or $B = 2\,A$, about 0.07 (see Table No. 1), or very much less than the average of those in America. From present knowledge it would seem, then, unwise to adopt in America the coefficients given in this table.

If the formula 7 500 $(1 + \varphi)$ is changed to 8 000 $(1 + \frac{7}{8} \varphi)$ then p, Mr. Cain. for the bridges mentioned, becomes nearly 0.184, as found by experiment, but the unit stresses vary but little from the previous values, and awkward fractions had best be eliminated.

Fig. 3 shows that the extreme unit stresses given by the Launhardt system agree sufficiently well with experiment, and it seems needless to lower the coefficient of φ unless it is deemed best to allow for a less number of repetitions than, say, 40 million, when, as stated before, the coefficient may be lessened somewhat.

The previous results were deduced primarily for a chord member.

The formula for unit stress = 7 500 $(1 + \varphi)$ is likewise applied here, empirically to some extent, to the web; only φ = Min. S ÷ Max. S, is found for the proper positions of live load giving Min. S and Max. S.

Thus for counters and middle ties, $\varphi = 0$ and $p = 0.33$, or somewhat greater than the 0.25 corresponding to average spans. This is certainly not to be regretted, as it would seem that all counters, whatever the length of span, should have the smallest unit stress allowed, not only because they are suddenly strained from zero to their maxima, but also on account of their small size, permitting greater lateral vibration, which must be accompanied by increased stress.

For intermediate ties and posts, the unit stress increases pretty regularly from the center to the ends, as it undoubtedly should.

In the formula for unit stress, 7 500 $(1 + \varphi)$, the coefficient 7 500 for double rolled iron in links or rods, is to be changed to 7 000 for plates or shapes (rolled iron) and to 6 500 for rolled iron in compression. Corresponding formulas for steel are used, and for compression members the usual reduction in unit stress should be made by the accepted column formulas.

It may be objected that not only are the values of p, in the table on page 532, too high, but also that they do not decrease rapidly enough as φ increases.

The objection is not so serious as it would at first appear, as is brought out more plainly by the author's Fig. 3 than could be done by any tables.

Here, for the Indian experiments, the straight line for the modified Launhardt formula does not depart sufficiently from the curve given by experiment to invalidate the result as a practical solution, and the same may be said for the author's formula, $7 - 4 R^2$, for safe unit stress in tons per square inch. The latter is a little nearer the truth, but it is a question whether the slightly greater accuracy cannot be sacrificed to gain the simpler modified Launhardt formula, particularly considering the very approximate nature of the solutions proposed for the various elements of the problem.

Mr. Moncrieff. J. M. Moncrieff, M. Am. Soc. C. E.—No more important subject for a paper, from the point of view of the bridge engineer, could be selected, but exception may be taken to the title, as the author deals exclusively with tensile stress, and the determination of safe compressive stress is entirely unnoticed, although compression members usually form a much larger percentage of the weight of a girder than the tension members. The highly important question of safe shearing stress on rivets subjected to variable loads is also untouched.

Whilst the weights of bridges of similar type certainly increase with the span, yet it is hardly correct to base thereon the assumption that the practice of bridge engineers in varying the unit stress according to the span is equivalent to an acknowledgment that the effect of moving load becomes relatively less as the fixed load becomes greater.

The importance of the span, rather than the weight of a bridge, in determining the probable influence of a live load receives recognition, according to the author's own references, in the practice of American engineers, under the special allotment system, in deciding on the allowable stress according to the span of the bridge, and the position of the member in the truss.

It is not at all definitely proved that "for any member of a bridge * * * the degree or percentage by which the effect of a ton of moving load is greater than a ton of fixed load varies for different ratios of moving load to fixed load," and proof is also required that "immediate" effect and "cumulative" effect can be in any sense looked upon as two distinct features of moving load influence on the materials of which a bridge is made, or, in other words, that dynamic influence and fatigue, in the usual acceptation of that term, have a definite separate existence. This is still an open question. With regard to the effect of horizontally moving loads on a bridge, in introducing the subject of Working Stress in Railway Bridges for discussion at one of the meetings of the Conference of the Institution of Civil Engineers, in London, in May, 1897, the writer pointed out that:

(1) "Although the influence of speed is difficult to estimate, yet *it does not follow that the maximum effect corresponds with maximum speed.* A familiar instance of this is found in the fact that a skater may frequently pass over ice that would not bear his weight for a single second if he were standing still, and, on the other hand, it is known that the maximum effect on an elastic structure does not occur when the load is standing still or going very slowly.

(2) "The deflection of a particular bridge under a train at 60 miles an hour may very possibly be no more, or even less, than when the train is going dead slow, and *there is probably, for any particular bridge, some particular speed at which a train will produce its maximum effect*, and this speed will evidently depend on the span of the structure, and it also appears quite possible that a train running at high speed may produce a relatively greater effect upon a bridge of considerable dimensions than upon one of short span."

Mr. Moncrieff.

This, of course, only refers to smooth running loads and is entirely exclusive of such influences as, (1) the action of the internal mechanism causing variations in the axle loads of locomotives; (2) the action of balance weights; (3) the dynamic effect of pitching and rolling of engines, arising partly from the two causes just mentioned.

These and other causes of additional stress require, and should be assigned, a special percentage of increase in effect as compared either with fixed or smooth running loads, or otherwise the same unit stresses for an electric or cable railway, as for the very different case of a railway with ordinary locomotives, with all the prejudicial influences of their internal mechanism, balance weights, pitching, rolling, etc., would have to be adopted.

The author's deduction that "the faster the engine (smooth running) moves in a horizontal direction, the less will be its effect in a vertical direction," is contrary to experience. If it were correct, horizontally running loads would be of less effect than fixed loads of equal amount as regards the production of deflection in a bridge or measurable strain in its members, whatever the relation between speed and span.

It is now also fully recognized by bridge engineers that it is absolutely necessary to keep in view the possibility and probability of trains running onto a bridge at great speed and being suddenly stopped on the bridge, and this is an occurrence which may happen any day on any bridge.

The author's difficulty in accepting the "live load" of the text books as assisting in the consideration of the influence of horizontally moving loads is somewhat difficult to understand, in view of the contingency just mentioned, and in view of the practically speaking rapid loading and unloading of the central diagonals of a lattice bridge under high-speed passing loads.

Referring to what may be termed the principal point of interest in the paper, *i. e.*, the deflection observations in India: In the first place, deflection observations on bridges with such proportions of depth to span most likely to be in use inIndia, but of which no particulars are given by the author, will no doubt give some idea of the effect of moving loads upon the flanges or booms, but such observations can hardly be taken as at all applicable to web members.

Attention also requires to be drawn to the fact that the "Standard Curve" given in the paper is an expression of averages only, and that for each point of the curve the maximum and minimum observations often differed widely from each other. It would be extremely interesting and valuable to have some idea of the actual variation.

Sir Benjamin Baker has very truly remarked that "it is the deviation from the average which really is so important in the design of engineering structures."

Mr. Moncrieff. It is not practicable to form any correct conclusions as to the influence of moving loads on bridges without full information as to span, type of bridge, proportions of bridge, character of details, speeds of trains, amounts of fixed and moving loads, and the full range of the results of the observations.

Again, referring to the division of moving load influence into "immediate effect" and "cumulative effect," as adopted by the author, it is by no means clear that the testing apparatus used by Wöhler, Spangenberg and Bauschinger (of similar type in each case) was such as to ensure that dynamic influence was completely eliminated, although shocks and jarring no doubt were avoided, but that is a very different matter. The writer is not aware that there are any records in connection with these tests of the "immediate" effect, *i. e.*, of the deflection or strain accompanying the stresses, and with the dimensions of the test pieces used it would be a very difficult matter to take observations of such small, but none the less all important, effects. An account of Wöhler's experiments, with illustrations of the testing machines, was published* in 1871, and a careful study of the machines and results of the tests affords good grounds for doubting the elimination of dynamic influence.

Under the head of "Results for Steel," the author states that for present-day mild steel of 27 tons tensile strength the elastic limit may be taken as high as 17 tons. This appears very doubtful. Does this figure refer to the primitive elastic limit of the material as it comes from the rolling mill?

One of the most important deductions to be drawn from Bauschinger's experiments is, that the primitive elastic limit of material, as it comes from the mill, is a most uncertain gauge of the powers of endurance of that material.

For instance, Bauschinger's endurance tests of wrought-iron plate, with a primitive elastic limit of 6.84 tons and ultimate strength of 25.2 tons, had powers of endurance approximately equal to that obtained with bar iron, having a primitive elastic limit of 11.8 tons to 14.8 tons and an ultimate strength of 26.7 tons per square inch.

Again, in one of Bauschinger's experiments on a Bessemer steel bar, he was able to cause the elastic limit to fluctuate all the way from 17.7 tons to 1.6 tons in tension, and from 9.65 tons to 3.24 tons in compression, by imposing loads alternately compressive and tensile, and the bar ultimately finished up with elastic limits equal for tension and compression, and amounting to 9.65 tons, which Bauschinger termed the natural elastic limit.

In comparing wrought iron and mild steel, it is absoluely necessary to have full details of their actual qualities, as the terms both cover material of a wide range of quality and physical character.

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* (London) *Engineering*, Vol. xi.

The endurance tests of mild steel and wrought iron by Bauschinger Mr. Moncrieff. and by Baker, on more modern material than that tested by Wöhler, agree in assigning a relative power of endurance to mild steel at least quite proportional to its greater strength, as compared with the wrought iron, and the latter was certainly of higher quality than would be generally used in ordinary plate or lattice girders, while the mild steel was of quite ordinary character. This is quite contrary to the deductions of the author.

Mansfield Merriman, M. Am. Soc. C. E.—The terms "immediate Mr. Merriman. effect" and "cumulative effect," as used by the author, involve essentially the same ideas as those expressed by American engineers in the words "impact" and "fatigue." "Impact" includes the increase in stress due to the velocity of application of the live load and also due to oscillations. "Fatigue" means the lowering of the ultimate strength by a very large number of applications of the live load. The latter occurs only when the elastic limit is exceeded, provided there is no alternation from tension to compression, while the former occurs under every intensity of stress caused by the live load. The author's combination of impact and fatigue under the term total effect is the only systematic one that has come to the writer's notice, and the tables presented in the paper will undoubtedly be of great value in all future discussions.

It is much to be desired that a detailed account of the deflection tests, from which the impact percentages were deduced, should be published. The increase in stress due to impact depends, not only upon the ratio of dead to live load, but also upon the length of the span and the velocity of the train; hence a discussion of these numerous experiments might add important information regarding the practical influence of these two elements.

The rules deduced by the author for the breaking stress, in tons per square inch are: $21 - 12 R^2$ for wrought iron, and $27 - 15 R^2$ for steel, in which R is the ratio of the live to the total load. The more usual method is to designate the ratio of the dead to the total load by ϕ. Thus $R + \phi = 1$, and hence $R = 1 - \phi$. Then the author's rules, expressed in terms of ϕ, are:

$$9 \left(1 + \frac{8}{3}\phi - \frac{4}{3}\phi^2\right) \text{ for wrought iron,}$$

$$\text{and } 12 \left(1 + \frac{10}{4}\phi - \frac{5}{4}\phi^2\right) \text{ for steel;}$$

and in these, $\phi = 1$ expresses the condition of all dead load, and $\phi = 0$ the condition of all live load, or in general $\phi = \dfrac{\text{Min. stress.}}{\text{Max. stress.}}$

The paper does not treat of alternating stresses where the minimum and maximum values are of different kinds, or where ϕ is negative. The rules are inapplicable to such cases, for, if the greatest tensile and

Mr. Merriman. compressive stresses are equal, then $\phi = -1$, and the breaking strengths given by the formulas are negative, or absurd. This appears to indicate that the rules are lacking in generality, for the perfect rules, when they are found, must be true for all values of ϕ from $+1$ to -1.

While the author's tables and rules are of great value for the case of the rupture of bridge members, under a very large number of repeated stresses which do not alternate from tension to compression, it may be doubted whether they retain this high value after division by the factor of safety of 3. Bridge members which are designed by the rules $7 - 4\ R^2$ and $9 - 5\ R^2$, for wrought iron and steel, respectively, are not stressed beyond the elastic limit, except in cases of accident; hence they are not subject to cumulative effect or fatigue, for all experiments appear to indicate that the properties of the metal are unimpaired by such stresses. In such cases, however, the stresses due to immediate effect, or impact, maintain their full influence. It appears, therefore, that the use of the author's formulas for the design of bridge members cannot be logically justified except as really introducing a factor of safety higher than the assumed 3. Until more perfect rules are established they should receive careful consideration, since they are a decided step in advance of many arbitrary methods now in use, and are apparently of greater generality than the formulas of Launhardt.

Mr. Wilson. JOSEPH M. WILSON, M. Am. Soc. C. E.—This paper is certainly a very interesting and practical one, directly in line with what is most needed in the determination and agreement upon a standard specification for bridges.

The application of systematic observations on deflections of bridges in India, under moving loads, to a comparison with methods of calculation already adopted or proposed, is most valuable.

While observed deflections cannot always be relied upon as an actual expression of the normal effect of moving load, the result being influenced by accidental or individual peculiarities of each particular structure, quality of workmanship, especially in riveted structures, etc., yet with such a number of observations at command, as in the present case, and the method of averaging adopted, they certainly form a fair, reasonable and most valuable addition to previous available data on this important question.

The systematic and exhaustive manner in which the author has carried out his investigations and comparisons gives great confidence in his results and leaves little to be desired.

It is quite evident that while one system of calculation, among those considered, may be deemed more comformable to the actual practical effects of the loading than another, and advisable to adopt for reasons of its own, yet, with the variations known to exist in the

best selected material, no one need feel uneasy with a bridge on account of its being calculated by either of these systems. Mr. Wilson.

The substitution of the term "moving" instead of "live" load, is a correction which should at once be adopted by every engineer, and the author is quite right in his explanation of the difference in meaning between these two expressions.

The writer looks with favor upon the "range" formula as offering advantages, in that it compels a lower working stress for short-span bridges where the fixed load is small as compared with the moving load, and permits a higher stress for longer spans where the reverse is the case. This is a matter in which, in the writer's opinion, the general practice will bear improvement, and some modification should also be made in the case of bridges near terminals, where the concentrated traffic of perhaps a number of lines accumulates, throwing on bridges an almost ceaseless service, the practical results of which, in America, at least, could not have been appreciated or anticipated a few years ago. America is advancing by vast strides, especially in its Eastern States, and that it is rapidly becoming an old country is shown in a marked way by the immense increase of local travel to and from its large cities.

J. B. Johnson, M. Am. Soc. C. E.—The author has conferred on the Society both a favor and a compliment by presenting this very thorough paper. Mr. Johnson.

As based upon the actual experiments of bridge deflections, his conclusions seem to be entirely justified. His use of a variable coefficient for reducing moving to an equivalent fixed load seems to be consistent with all known observations, and, as shown by Fig. 3, this use warrants a working stress for the ordinary ratios of fixed to moving loads some 25% greater than the assumption commonly made in the United States, which is that this coefficient should have the constant value of 2. His formula is simpler, also, and it may be written as follows:

$$p = \frac{f}{K}\left(1 - \frac{R^2}{2}\right) = \frac{f}{6}\left(2 - R^2\right) \dots\dots\dots\dots\dots (1)$$

where $p =$ total working stress per square inch,

$f =$ ultimate tensile strength of the metal,

$R =$ ratio: $\frac{\text{live-load stress}}{\text{total stress}}$ in any member,

$K =$ factor of safety.

When the American formula (with the constant factor 2 to reduce live to dead loads),

$$\text{which is } p = \frac{a}{1 - \frac{\text{Min. stress}}{2\text{ Max. stress}}}$$

is put in the same form as equation (1) it becomes,

$$p = \frac{f}{K}\left(\frac{1}{1 + R}\right) \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

Mr. Johnson.

As the author's formula (1) is at once more rational, or at least more in accordance with observed facts, and also gives about 25% greater working stresses, there would seem to be every reason why it should be used.

Mr. Stone.

E. HERBERT STONE,* M. Am. Soc. C. E.—Before replying to the discussion the author wishes to thank the members for their very kindly and courteous reception of his paper. The subject is one which American bridge engineers may justly claim to have studied in all its bearings more thoroughly, more practically and more successfully than any other body of engineers in the world, and it is therefore specially gratifying to the author to find his contribution so favorably received.

In condensing the matter within reasonable limits for a paper, it was unfortunately necessary to leave out much in the way of explanation which would have anticipated some of the points raised in the discussion. The author's original work, however, is in the hands of many of the members, and a copy is in the library of the Society.

System of Adjustment.—It is now generally admitted that the effect of the Moving Load on a member of a bridge is more severe than that of the same weight as Fixed Load. It is further admitted that the degree or percentage by which the effect of Moving Load is thus greater than that of the same weight as Fixed Load is not in all cases the same, but becomes greater as the percentage of Moving Load in the Total Load increases. It is also agreed that for each particular case, if all the conditions could be ascertained and allowed for, there would be found a certain coefficient, by which, for practical purposes, the nominal Moving Load might be multiplied to obtain its equivalent or effective stress as Fixed Load.†

Opinions differ, however, as to the system under which this coefficient should be determined. Some engineers maintain that it should depend on the span of the bridge, while others are of opinion that it should depend on the proportion of Moving Load in the total compound load.

In this connection it may be pointed out that the system of making the coefficient vary with the span of the bridge would, if logically followed, assign the same unit stress to every member of the same bridge, irrespective either of its position in the structure, or of the ratio of Moving Load to Fixed Load. It is clear, however, that this system can only be considered approximately correct for the main

* The author has also forwarded supplementary discussions, designed to still further elucidate the paper, on "The Effect of Alternating Stresses," "The Factor of Safety," "The Trajectory Effect," "The Effect of Varying the Conditions," and "The Nature of Experiments Required"; all of which are filed for reference in the library of the Society.

† The author is aware that, on theoretical grounds, some engineers hold that the dynamic effect of the Moving Load cannot be represented properly by the static effect of a Fixed Load. For the practical purposes of bridge design the varying effects of the two kinds of load must be provided for on some definite system, and it is only a question of how this may most conveniently be done with sufficient accuracy for the purpose required. The practical side of the question is very clearly stated by Mr. J. A. L. Waddell in the first two paragraphs of his discussion.

booms—it is less suitable for the web bracing—and cannot apply at all to the cross-girders and rail-bearers.* Mr. Stone.

A rule based on this system must, moreover, often give inconsistent results, inasmuch as for bridges of the same span not only does the actual weight differ, but the relative weight of the Fixed Load as compared with that of the Moving Load depends greatly upon other conditions, such as gauge, the number of lines provided for, the amount of ballast, nature of flooring, and design of the structure generally.

It is also to be noted that a member of a small-span bridge may be under precisely the same conditions of loading as a member (either in the same or in some other position) on a large-span bridge, and in such case the size of the whole structure evidently does not at all affect the stresses to which that member is subjected.

It has further to be remembered that, of the total effect to be provided for, the cumulative effect is of very great importance; and it would be somewhat difficult to apply the results obtained by Wöhler and others for repetition and alternation of stress to a system under which the permissible unit stress is made to vary with the span.

There can be no doubt that to contend that the permissible unit stress should vary with the span is practically an admission of the principle that the permissible unit stress should vary with the ratio Fixed Load to Moving Load. For, if examined, this contention really means that in larger spans the mass of material in the structure itself is relatively greater as compared with the mass of material in the moving locomotive, and the effect of the locomotive in producing violent shocks and damaging vibrations is thereby to a greater extent modified, reduced or dispersed.

If an anvil be struck by a hammer, and the weight of the anvil as compared with that of the hammer be great, the amount of movement and vibration produced in the anvil will be relatively small. Further, if experiments were to be conducted with a series of hammers and anvils—so arranged that for each hammer and its anvil the relative weight differed, while in each case the total weight of the hammer plus its anvil remained the same—then it is evident that (other things being equal) the actual amount of movement and vibration would be greatest in that case for which the fraction $\frac{\text{Weight of anvil}}{\text{Weight of hammer}}$ was least.

On these considerations the author has preferred to make the variation in the coefficient depend on the proportion of Moving Load in the total compound load. He believes that this system is now approved by the majority of bridge engineers, and is adopted of necessity by all

* The cross-girders and rail-bearers in a long-span bridge would, of course, be treated as separate bridges of short span, with correspondingly lower unit stresses.

Mr. Stone. engineers who use Wöhler's results as embodied in the formulas of Launhardt or others. This system is, moreover, of universal application, whereas the system of variation by span applies to bridges only.

Fixed Load.—Where there are alternating stresses it is to be noted that a portion, or in some cases the whole, of the load due to the structure itself may become Moving Load, *i. e.*, a load which is applied and removed each time a train traverses the bridge. The Fixed Load to be dealt with in such cases is, therefore, not the actual total Fixed Load due to the structure, etc., but the residual Fixed Load, which remains unaffected by the application of the Moving Load, and which is therefore not subject to alternation.

Moving Load.—Mr. F. E. Turneaure has apparently misunderstood the author's allusion to the "Live Load of the text books." This special sort of "Live Load" is a load suddenly applied in a vertical direction, but without impact. Its immediate effect is exactly double that of an equal weight as "Dead Load." Regarding this the following extracts are quoted from well-known authors:

"Theoretical considerations, at first sight, might erroneously lead us to anticipate a greatly increased deflection under a rolling load, since they indicate the maximum deflection of an elastic beam under a suddenly imposed load to be double that of the final deflection after oscillations have ceased. The reason for this is obvious enough, for the resistance of the beam in deflecting to a certain extent, measured in foot-tons, must be equal to the work done by the load in its descent to the same extent; and as the latter is equal to the product of the load into the ultimate deflection, the former must be equal to the product of the mean resistance of the beam into the same deflection; hence the mean resistance of the beam must be equal to the load; and as the resistance is *nil* at the commencement of the bending, it must be equal to double the load at the termination" (B. Baker—"Long and Short Span Railway Bridges"—Edition 1873, page 92).

"It is well known that a load when suddenly imposed will produce a momentary deflection twice as great as that due to the same load at rest.

"This very important fact is illustrated every day in the action of an ordinary spring balance. Thus, if we take a common letter-balance, selecting one which works with very little friction, and if we suddenly place upon it (without shock or impact) a letter weighing 2 ozs., we know that the index-finger will at once be driven down the graduated scale until it points to something like 4 ozs., although after a few vibrations it will of course come to rest at the figure indicating the 'correct weight' of the letter. The extreme position of the index-finger upon the scale shows that the maximum stress, as well as maximum deflection of the spring, is twice as great as would be due to the same load at rest; and in the case of an elastic girder, a column, or a suspending rod, the same thing must take place irrespective of the amplitude of the vibrations, which in these cases may be so small as to escape observation." (T. Claxton Fidler.—"A Practical Treatise on Bridge Construction"—Edition, 1893, page 242).

These quotations will, no doubt, make more clear the meaning of the author's remarks under "Live Load contrasted with Train Load." Mr. Stone.

The author sees no reason to alter his opinion that, "if we neglect the effects of vibration and deflection* and suppose the engine to run with absolute smoothness, it is clear that the faster the engine moves in a horizontal direction, the less will be its effect in a vertical direction." The illustration of the skater on thin ice will be familiar to every one. If the skater, having got up a high speed, puts his feet together and allows himself to glide on smoothly, he can traverse safely over ice which would give way at once under his weight if he stood still, and the faster his motion in a horizontal direction the less is the effect on the ice of his weight in a vertical direction. In other words, the effect of the weight of the skater on the ice is reduced instead of being increased by the speed at which he travels. If, however, instead of a smooth, gliding movement we have vertical oscillation, the conditions are much altered, and we find that ice which would just bear the weight of a man standing quietly upon it, would give way at once if he attempted to run on it, and the more violently he moved his feet and legs in running, the greater would be the effect in breaking the ice.

The point would be of no importance if it were not that some persons still believe that the "Live Load" of the text books corresponds in some way with the Moving Load of a locomotive, and it is quite common in older text books to find the statement that the effect of "Live Load" is just double that of the same weight as "Dead Load."

Two-Fold Nature of the Effect.—Mr. J. M. Moncrieff considers that proof is required that "immediate effect" and "cumulative effect" can be in any sense looked upon as two distinct features of Moving Load influence.

Mr. F. E. Turneaure, on the other hand, remarks as follows:

"The author has done the profession a great service in separating and discussing so thoroughly the two effects of the moving load. These effects are too often treated apparently as one, but the distinction between them needs to be kept clearly in mind in discussing the subject of working formulas."

Mr. Mansfield Merriman† says:

"I have read the paper with much interest and regard it as a valuable contribution to this important subject. Your distinction between 'immediate effect' and 'cumulative effect' is in particular calculated to simplify the theoretic discussion of dynamic action."

* The deflection of the girder introduces an entirely new element, namely, the "trajectory effect." It will be evident on consideration that this element in its action and mode of application is essentially different from the theoretical "live load" of the text books. To take the illustration of the skater, the nature of the trajectory effect would be well represented by the skater gliding at a high speed over the ice and suddenly falling on his knees. The trajectory effect is of the nature of a blow which may under certain conditions be very severe. In the experiments conducted at Portsmouth in 1848, under the direction of Commissioners Appointed to Enquire into the Application of Iron to Railway Structures, results for the trajectory effect upwards of three times as great as that due to the same load at rest were frequently obtained, and the highest recorded result was more than five times as great as that of the same load at rest.

† Letter to the author, dated January 25th, 1896.

Mr. Stone. *Immediate Effect.*—The method by which the "immediate effect" was deduced from the Indian experiments is described in the paper. The mass of detail from which these results were obtained was very great, and much of this material is no longer accessible to the author. The final plottings before the average curve was drawn are, however, still available, and can be sent to any one interested. The drawings have been shown to several engineers, who agree that the curve as reproduced in Fig. 1 is probably as good a representation of the results as could be obtained. The speed may be taken as about 40 miles an hour.

Mr. J. A. L. Waddell considers that the difference in design between Indian and American bridges might render it doubtful whether the results could be made applicable to American bridges. It would appear, however, that a rule under which the effective load on a member is made to depend on the proportion of fixed load and moving load would apply equally well to different designs, or at any rate sufficiently so for practical purposes.

Mr. H. S. Prichard notes that the curve for "immediate effect" changes its direction and assumes a different character for the higher ratios of Moving Load. This feature is due to "trajectory effect." What is believed to be the normal position of the curve is shown by dotted lines in Fig. 1.

The author ventures to think that Mr. William Cain has misunderstood the mode in which the results were worked out. The Indian experiments were by deflection only, and the girder was in each case treated as a whole. The results obtained from any particular girder were not considered applicable to every member of that girder, but to any member (either in that or in any other girder) in which the ratio $\frac{\text{Fixed Load}}{\text{Moving Load}}$ was the same. For example, suppose in a particular class of girders the ratio $\frac{\text{Fixed Load}}{\text{Moving Load}} = \frac{20}{80}$, and that for these girders the experiment showed that the immediate effect of the Moving Load was 113% of that due to the same weight as Fixed Load. This 113% "equivalent" would be considered as suitable to any member for which the same ratio $\frac{\text{Fixed}}{\text{Moving}} = \frac{20}{80}$ applied, but not necessarily to any individual member of the particular girder tested.

The author, however, does not attach any particular importance to the Indian experiments, and has only made use of them in his paper because there were no others of such an extensive nature available. He wishes them to be considered merely as giving general average results and will gladly see them superseded later on by a better series of experiments made on scientific principles by skilled observers.

Results for Steel.—It has been remarked in the discussion that the greater part of the author's paper is devoted to structures of wrought

iron, and that only a comparatively small portion is occupied with the results for steel. The reason for this will be obvious on consideration, as the greater portion of the argument applies both to iron and to steel and could be used for either material indifferently by a mere alteration of the figures representing the stresses and coefficients. It was therefore considered better to let the argument run through continuously for one material, *viz.*, wrought iron, and then show how the results thus obtained should be applied to steel. The only alternative to this method of dealing with the subject would have been to have given the greater portion of the paper twice over. Mr. Stone.

Some engineers consider that the author in taking the elastic limit of steel as used for modern bridge work at two-thirds the ultimate breaking stress has taken a higher ratio than would ordinarily be obtained in practice. In this connection the following is quoted from *Engineering News*, February 28th, 1895, page 135:

"It has frequently been claimed by structural steel makers that in drawing specifications the elastic limit should not be specified to exceed half the ultimate strength, and that at least 10 000 lbs. variation should be allowed between maximum and minimum limits of ultimate strength. Some recent tests, however, have shown that there is no difficulty in procuring steel with an elastic limit equal to two-thirds the ultimate strength, and that a difference of 7 000 lbs. between the specification limits for ultimate strength can be met without difficulty. We are indebted to Mr. George H. Thomson, M. Am. Soc. C. E., Consulting Engineer, for the accompanying condensed record of seventy-one tests upon 600 tons of bridge steel made to his specifications and rolled in January, 1895, by the Pennsylvania Steel Company and Carbon Steel Company and inspected by Mr. A. C. Cunningham, of the American Engineering and Inspection Association, of Albany, N. Y. The steel consisted of plates and angles $\frac{3}{8}$ in. to $\frac{3}{4}$ in. thick, for use in six spans on the Central Vermont Railroad, and no difficulty was found in meeting the high requirements of the specifications, only a very few pieces tested falling below the requirements. The lowest elastic limit found was 61.1% of the ultimate strength. The condensed record of the tests is as follows:"

ABSTRACT OF RESULTS.*

	Static breaking stress. Tons.	Elastic limit. Tons.	Elastic limit ratio. Tons.
Pennsylvania Steel Co.			
With lowest elastic limit	26.295	16.571	63.02
With greatest ultimate strength	28.362	18.147	63.98
With least ultimate strength...	25.491	17.424	68.35
Carbon Steel Co.			
With maximum elastic limit...	25.518	19.411	76.07
General Results.			
Highest results	28.362	19.411	68.44
Lowest results................	25.491	16.571	65.01
Average	27.057	18.108	66.93

* In the original the stresses are given in pounds, and have here been converted to their equivalents in tons for convenience of comparison with other values given in the paper. The table given here is a condensed abstract of that given in *Engineering News*.

Mr. Stone. Hence it would appear that for the quality of mild steel, as now used for the construction of railway bridges, if the ultimate breaking stress per square inch be taken as 27 tons, the elastic limit may for practical purposes be taken as high as 18 tons; and, in dealing with cumulative effect the rule for safe working stress may then be based on a value of u equal to $\frac{2}{3}\,t$.

Members in Compression.—If the effect of bending be sufficiently provided for by the design and by the use of a suitable column formula, there appears to be no reason for adopting a lower permissible working stress for members in compression than for those in tension. The fact that the ultimate breaking stress in tension may be somewhat higher than the ultimate crushing stress in compression is here not of much importance compared with the fact that (if the effect of bending be provided for), we may, for evident physical reasons, work to a higher unit stress in compression than would be safe or prudent in tension. For a member in compression the "cumulative effect" is a matter of comparatively small importance, and many of the reasons for adopting a factor of safety as high as 3.0 for members in tension do not apply to members in compression. There can be no doubt that, if the bending effect be adequately and separately provided for, the material in compression may safely be subjected to a greater working load than would be admissible in tension.

Application of Results.—When the permissible stress under different conditions has been determined, the question at once arises as to the method by which the results thus agreed upon shall be applied in practice.

Take for example the case of a member of a truss of wrought iron conditioned as follows:

	Tons.
Fixed Load	20
Moving Load	80
Total	100

and let it be assumed that the following have been ascertained or decided upon as suited to these conditions—

	Tons per square inch.
Safe Working Stress:	
For all Fixed Load	7.00
For the compound (nominal) load	4.44

and that the coefficient is 1.72.

It is evident that we may proceed to ascertain the area required for the member either by the effective stress obtained by the use of the coefficient thus:

Mr. Stone.

Effective Stress Method.

Effective stress $20 + (80 \times 1.72) \ldots = 157.6$ tons.

Safe working stress per square inch = 7 tons.

Area required $\frac{157.6}{7}$ .... = $22\frac{1}{2}$ sq. ins.

or from the nominal stress thus:

Nominal Stress Method.

Nominal stress $20 + 80$ .... = 100 tons.

Safe working stress per square inch = 4.44 tons.

Area required $\frac{100}{4.44}$ .... = $22\frac{1}{2}$ sq. ins.

Under the Effective Stress method the same safe working stress (*viz.*, 7.0 tons per square inch) would be used for every member of the bridge and applied to the effective stress. The coefficient would be obtained from a table, or from a diagram similar to Fig. 2.

Under the Nominal Stress method the permissible nominal stress per square inch would vary, and would be obtained from a table, or from a diagram similar to Fig. 3.

In practice it is probable that in either case the rule or formula would in the first instance be used to construct a table or diagram, and that such table or diagram would for general work be used in preference to the formula.

The Effective Stress method appears to be not only the more scientific, but is also more convenient, as has been very clearly shown by Mr. H. S. Prichard.

Mode of Calculation.—Mr. J. L. Power O'Hanly has raised some questions regarding the mode of calculation adopted.

Table No. 1.—Taking for example the case cited by Mr. O'Hanly in which the percentage ratio is $\frac{\text{Fixed}}{\text{Moving}} = \frac{2.5}{97.5}$ and suppose for the sake of illustration that these figures represent tons, we have then in this case a loading as follows:

	Tons.
Fixed Load	2.5
Moving Load....................................	97.5
Total load.....	100.0

It is found by experiment that for this ratio of loading the immediate effect of the Moving Load is 1.474 times as great as that due to the same weight as Fixed Load.

Hence the equivalent of the Moving Load in terms of Fixed Load is

$$97.5 \times 1.474 = 143.715 \text{ tons;}$$

and the total effective load is

Actual Fixed Load.		Equivalent Moving Load.		Total Effective Load.
2.5	+	143.715	=	146.215 tons.

Mr. Stone. That is to say, the 100 tons of load, made up of 2.5 tons fixed and 97.5 tons moving, is equivalent, as far as stresses on the structure are concerned, to 146.215 tons all Fixed Load. Taking 21 tons per square inch as the breaking stress for all Fixed Load, the breaking stress per square inch for this Nominal Load of 100 tons will be $\frac{21 \times 100}{146.215}$ $= 14.3624$ tons per square inch.

Taking a factor of safety of 3.0 we get permissible working stresses as follows:

	Tons per square inch.
For the Effective Load	7.0000
For the Nominal Load	4.7875

and either $\frac{146.215}{7}$ or $\frac{100}{4.7875}$ will give the area required, *viz.*, 20.89 sq. ins.*

As regards the apportionment of the breaking stress (taking the same case) we have ascertained that a bar 1 sq. in. in section will, under the conditions assumed, break with a Nominal Load of 14.3624 tons. This load is made up of 2.5% Fixed Load and 97.5% Moving Load. Hence the Nominal Load which will break the bar of 1 sq. in. in section is made up as follows:

	Tons.
Fixed Load	0.3591
Moving Load	14 0033
Total	14.3624

But to obtain the actual effective load we must multiply the nominal moving load by the coefficient 1.474, and we then get the following as the actual effective load which will break the bar of 1 sq. in. in section:

	Tons.
Actual Fixed Load	0.3591
Effective Moving Load	20.6409
Total	21.0000

Tables Nos. 2 and 3.—A similar explanation will apply to Tables Nos. 2 and 3 for the cumulative effect of the Moving Load as deduced by the use of Gerber's parabola and Launhardt's formula, respectively. The results have all been worked out to four places of decimals, and the nearest figures to two places of decimals have been entered. The figures have been re-checked by the author and found correct.

Table No. 5. Bell-Robertson Rule.—In regard to the results given in Table No. 5 for the coefficients for the Bell-Robertson rule the process of calculation is as follows:

* It is to be remembered that these figures are for "immediate effect" only. Cumulative effect is not here taken into account.

Taking the example selected by Mr. O'Hanly, *viz.*—$\frac{\text{Fixed}}{\text{Moving}} = \frac{2.5}{97.5}$; Mr. Stone.
suppose the total load to be 100 tons.

Then under the rule we have:

	Tons.
Moving Load (97.5) multiplied by 1.5..........	= 146.25
Add a minimum Fixed Load equal to half the Moving Load $\frac{97.5}{2}$........................	= 48.75
Total Effective Working Load................	= 195.00
Deduct actual Fixed Load....................	2.50
Effective Moving Load.......................	= 192.50

The coefficient for obtaining the equivalent Effective Load due to the nominal Moving Load will therefore be $\frac{192.50}{97.5} = 1.9744$.

Table No. 5. Launhardt's Formula—The remarks about the law of the variation of the coefficient for the modified Launhardt formula will, perhaps, be made more clear if expressed as a formula thus:

$$\text{Coefficient} = 1 + \frac{\text{Total}}{\text{Fixed} + \text{Total}}.$$

This is, however, merely a matter of general interest, as the coefficients given in the table were calculated independently, not by this rule.

Range Formula.—Taking the example selected by Mr. O'Hanly, the mode of calculation is as follows:

	Tons.
Fixed Load..................................	2.5
Moving Load.................................	97.5
Total..................................	100.0

Here $R = 0.975$ and $R^2 = 0.9506$.

By range formula: Breaking stress $= 21 - (12 \times R^2)$

$$= 21 - (12 \times 0.9506)$$
$$= 21 - 11.4072$$
$$= 9.5928$$

That is to say, that under the conditions assumed, a bar 1 in. square will break under a nominal load of 9.5928 tons made up as follows:

	Composition of Compound Load. Percentage.	Tons.
Fixed Load........................	2.5	0.2398
Moving Load.......................	97.5	9.3530
Total........................	100.0	9.5928

Mr. Stone. But the breaking stress for this same bar 1 in. square, under all Fixed Load, would be 21 tons.

Hence the nominal Moving Load of 9.3530 tons under these conditions produces an effect on the bar as severe as that due to 20.7602 tons of Fixed Load, thus:

	Tons.
Actual Fixed Load	0.2398
Effective Moving Load	20.7602
Total	21.0000

Under these conditions, therefore, the coefficient to be applied to the Moving Load to obtain its equivalent in terms of Fixed Load will be—

$$\frac{20.7602}{9.3530} = 2.2196$$

Selection of a System.—For the reasons given in the paper, the author believes that the range formula gives a closer approximation to the actual facts than any other of the six systems which have been discussed.* In form and principle it may be compared with the modified Launhardt formula, but it will probably be agreed, on an inspection of Fig. 3, that the curve given by the range formula is more satisfactory than the straight line of the modified Launhardt formula.

If, however, something simpler be preferred, the author would invite attention to the range coefficient system. It is exceedingly simple to use in practice, easily remembered, and, next to the range formula, gives better results than any other system. It will be remembered that under this system the coefficient is obtained by merely adding 1.0 to the decimal giving the proportion of Moving Load to Total Load.

Examples :

Moving Load 0.9 of Total Load.
Then coefficient = 1.9.
Moving Load 0.66 of Total Load.
Then coefficient = 1.66.

The author notes that Mr. H. S. Prichard considers this (the range coefficient) the best system that has been discussed.

Application to American Practice.—Mr. J. A. L. Waddell finds the results for permissible nominal stress as given in the paper higher than he would consider advisable to adopt. On the other hand, Mr. O. Chanute, on behalf of himself and Mr. C. L. Strobel, remarks:† "We are both much pleased with your work, and find that it differs but little from current American practice."

* It is probably in fact a closer approximation than even the results given by experiment, because it makes a certain allowance for the effect of vibration and concussion.

† Letter to the author, dated June 20th, 1896.

After examining a number of modern American specifications the Mr. Stone.
author would suggest for consideration the following modifications of his range formula as likely to be found suitable in general American practice.

Wrought Iron.

Pounds per square inch.

$$\text{Safe Working Stress} = 16\,000 - (9\,000 \times R^2)$$

Medium Steel.

$$\text{Safe Working Stress} = 20\,000 - (11\,000 \times R^2)$$

Should the results obtained by these formulas, when plotted, be found not to be quite satisfactory a revised line can then be sketched in to pass through the points required and an amended formula made accordingly.*

Modifications in the Formula.—The author wishes it to be understood that he does not attach any special importance to the precise figures given with his proposed range formula. This formula has been merely suggested by him as a means of obtaining a curve (see Figs. 2 and 3), which will represent the known facts better than those obtained by other rules which have been proposed.†

In any case, however, it will no doubt be agreed that the right principle is first to draw a curve in accord with the facts of the case as far as can be ascertained, and then to devise a formula which will represent this result as simply as may be and with a sufficient degree of accuracy for practical purposes.

Experiments Required.—With the engine standing at rest there is no difficulty in ascertaining the stress which should come on each member of the bridge owing to the weight at the wheel-tread of each wheel; and although we are aware that (owing to peculiarities of construction or to improper treatment during erection), each member does not, as a matter of fact, receive just the exact stress due to it by calculation, yet we are satisfied that for all practical purposes the results given by calculation are a sufficiently near approximation. But when we have to determine the special extra effect on each member of the bridge due to the condition of the load being changed from "engine-at-rest" to "engine-at-full-speed," the problem at once becomes exceedingly complex. Among the elements which affect the result, we have, for example:

(*a*) Temporary excess loads for particular wheels due to the pitching of the engine.

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* It will be observed that these formulas are based on the following assumptions:

	Wrought Iron per square inch.	Medium Steel per square inch.
Breaking Stress.....	48 000 lbs. = 21.43 tons.	60 000 lbs. = 26.79 tons.
Safe Working Stress	16 000 " = 7.14 "	20 000 " = 8.93 "

Tons per square inch.

† For wrought iron, the formula, Safe Working Stress $= 7 - \left(\frac{4R + 6R^2}{3}\right)$ gives very good results, but is more complicated.

Mr. Stone. (*b*) Alternating excess loads for particular wheels due to the pressure on the upper or lower guide bar, the motion of side rods, balance weights, etc.

(*c*) Lateral concussions of the wheel flanges against the side of the rail head.

(*d*) Effect of relation between wheel spacing, length of bridge panel and length of rail.

(*e*) Irregularities, local weakness, and other imperfections in the track.

(*f*) Deflection of the main girders (and also, in some degree, of the cross-girders and rail-bearers) in relation to the curve of trajectory of the engine.

(*g*) Wave period of the bridge in relation to the alternation period of the engine.

(*h*) Excess stresses induced by certain members not doing their proper share of work. Local bending or twisting stresses at joints and other stresses caused by imperfections in design, construction or workmanship.

(*i*) Irregular expansion or contraction due to changes of temperature having special local effects.

All these factors and others which might be specified, are modified in different degrees by the speed at which the locomotive travels, the design and dimensions of the bridge, the position of the member in the bridge, and the design and dimensions of the engine. Each factor is, moreover, independently variable, and in its variations affects most of the other factors to an extent which is not determinable by any mathematical process, causing in the other factors enhanced effects in some cases and diminished effects in others.

It is evident that for each particular case having its own definite conditions, the problem might be dealt with by two different methods, *viz.:*

(*a*) The analytical method. Under which each separate factor, so far as could be known or ascertained, would be examined and valued, and the net result obtained by a summation of these values*—and

(*b*) The experimental method. Under which the net result of all the stresses acting on the bar, due to all causes whatsoever, whether known or unknown, would be obtained at once as a total by means of a strain indicator.

The experimental method for obtaining the net immediate effect of the Moving Load is analogous to the use of a dynamometer for ascertaining the net tractive force required to haul a train. All elements of train resistance, whether known or unknown, are automatically

* Probably the best example of the analytical method is the paper by Professor Melan published in the *Journal* of the Austrian Engineers and Architects' Association in 1893. An abstract of this interesting paper is given in the *Proceedings* of the Institution of Civil Engineers (London), 1893-94, Vol. cxvii, p. 411.

Mr. Stone.

summed up under the net total exhibited by the dynamometer. The experimental method may not be theoretically accurate, but most engineers will agree that for practical purposes it is sufficiently satisfactory.

On the experimental method, when a sufficiently wide range of experiments had been made, a series of coefficients would be obtained representing (for the class of structure dealt with), the enhanced effect of Moving Load as compared with the same weight as Fixed Load under varying conditions. The coefficients thus obtained would be used as the basis for a formula or diagram for future use in the determination of the Safe Working Stress or Equivalent Effective Stress on similar structures.

As regards the nature of experiments required, the author would invite attention to the remarks made by Mr. J. A. L. Waddell in 1892,* and also in the present discussion. The author entirely agrees with Mr. Waddell, and hopes that before long a really good series of experiments in the direction indicated may be undertaken by skilled observers on a properly organized system.

When this has been done, and the results analyzed and compared, it may be found advisable to adopt a different series of coefficients for each different design of bridge, or for each different class of members; and any desired degree of refinement in rules or formulas may then be adopted. Meantime the author ventures to hope that the average results already obtained and published in this paper may be found of use to the profession as indicating approximately the general conditions and the nature of the effects to be provided for, and as providing a sufficiently good basis for an approximate rule or formula which may be elaborated and improved when better data become available.

* "Discussion on Railway Bridge Designing," *Transactions*, Am. Soc. C. E., 1892, Vol. xxvi, pp. 276 and 277.